Image sharpening by grid warping with curvature analysis

Andrey Nasonov, Andrey Krylov
Laboratory of Mathematical Methods of Image Processing
Faculty of Computational Mathematics and Cybernetics
Lomonosov Moscow State University
Moscow, Russia
nasonov@cs.msu.ru

***Abstract*—The paper proposes an improvement of the grid warping algorithm for solving the edge sharpening problem. The idea of the grid warping is to transform the neighborhood of the edges in order to make the edge transient area thinner. This approach does not amplify the noise and does not introduce ringing artifact. The idea of the improvement is to analyze the curvature of the gradient field at edge point and adjust warping vectors.**

***Index Terms*—Image deblurring, edge sharpening, grid warping, curvature analysis.**

I. Introduction

Image deblurring has been one of the challenging image processing problems for a long time. It is typically posed as finding a sharp image I_0 from the blurred observation I_B according to the blur model

$$I_B = I_0 * H + n \tag{1}$$

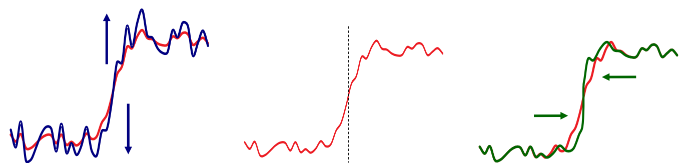
where H is the blur kernel and n is additive noise.

The problem (1) is ill-posed and cannot be solved directly. Regularization-based algorithms [1] provide an effective solution for this problem if blur kernel H and noise n are given with sufficient accuracy. Usually these algorithms have parameters that set a balance between sharp result with high probability of noise amplification and introducing ringing effect and smooth result with some blur remaining but with low probability of artifacts.

In real applications, the blur kernel and information about the noise type and level are not known and have to be estimated. There exist some approaches for blind image deconvolution [2, 3], but they are effective only for images with uniform blur and noise. The non-uniformity of image blur, errors in blur kernel and noise estimation negatively affect the quality of the result.

Another approach for image deblurring is using grid warping algorithm to sharpen the edges by making the edge transient areas thinner (see Fig. 1). The work [4] has introduced the warping approach for image enhancement by representing it as a solution of a differential equation. The solution of the equation is used to move the pixels from the edge neighborhood closer to the edge while stretching the areas between edges. The method has several parameters with difficult choice of the optimal values. Also local warping constraints affect the image globally resulting in distortion of image structures. In [5] warping vectors are calculated directly using left and right derivative values. In both methods [4] and [5], the warping vectors are proportional to gradient magnitude. It results in different sharpening strength for low- and high-contrast edges even if they have the same blur level. The warping can be expressed as morphology-based sharpening [6] and shock filters [7, 8, 9]. But these methods make the image appear piecewise constant which is effective mostly for cartoon-like images.

The work was supported by Russian Science Foundation grant 17-11-01279



(a) Typical image enhancement approach (b) Edge profile (c) Warping approach: pixels are shifted

Fig. 1: The idea of edge sharpening by grid warping.

The warping algorithm proposed in the papers [10, 11] eliminates the problems of the algorithm [4] by posing another differential equation and proposing a direct solution. The algorithm introduces a displacement function — a function that directly defines the shift vectors for one-dimensional edge profile. For each image pixel the warping vector is calculated as a weighted sum of displacement vectors for the edge profiles over the surrounding edge pixels. The work [12] improves the result of this algorithm by optimizing the displacement function that correspond to real optic blur. Also the implementation is robust to blur level estimation.

Compared to the classical image deblurring approach (1), the grid warping approach does not change the noise level and does not introduce ringing artifact. At the same time, sharpening by grid warping is applicable only to the edges while textured areas are kept intact. Therefore, the grid warping cannot be used as a standalone image deblurring algorithm but it is effective as a post-processing algorithm after existing image deblurring methods [12].

In this paper, we present an improvement of grid warping algorithm proposed in [12] by paying special attention to corners. The edges with sharp corners are displaced during blur but their position is not fully restored by image deblurring algorithms. We analyze the curvature of the gradient field in edge areas and perform additional warping to make the corners sharper.

II. Grid warping

In this section we describe the idea of edge enhancement using pixel grid transformation [12].

A. Single edge sharpening

Firstly, we consider the model with single straight edge in the image. In this case, the warping vectors are orthogonal to the edge and their magnitude is defined as a function of distance to the edge called the *displacement function*.

For any edge $g(x)$ centered at $x = 0$ its sharper version $h(x)$ can be obtained by shifting the pixels from edge neighborhood towards the edge. The displacement function $d(x)$ describes the shift of a pixel with coordinate x to new coordinate $x + d(x)$ [11]:

$$h(x + d(x)) = g(x).$$

The following restrictions are applied to the displacement function:

1. The transform should be monotonic, i.e. the relative order of pixels should be kept. It can be expressed as the displacement function constraint:

$$d'(x) \geq -1. \quad (2)$$

2. The areas far from the edge should not be affected by grid warping:

$$d(x) \to 0, \qquad |x| \to \infty.$$

The choice of the displacement function $d(x)$ influences the grid warping output. On the one hand, the edge should be as thin as possible. On the other hand, there should not be gaps between adjacent pixels due to stretching. Also the displacement function should correspond to the blur type. The paper [12] suggests using the following displacement function for common types of optic blur:

$$d(x)[a] = \begin{cases} -x, & |x| \leq a, \\ \frac{3a-2|x|}{a} \operatorname{sign} x, & a < |x| \leq \frac{3}{2}a, \\ 0, & |x| > \frac{3}{2}a, \end{cases}$$

where a is the parameter proportional to blur level.

B. Grid warping for 2D images

The papers [10, 11] describe how grid warping for edge sharpening is applied for 2D images.

1) Single-edge algorithm: The simplest algorithm consists in finding the nearest edge for every pixel and applying the one-dimensional algorithm using that edge (see Fig. 2). It includes the following steps:

1. Estimate the blur level for the edges.

2. For all pixels in the neighborhood of the edge compute the distance x to the nearest edge point.

3. Calculate the displacement value $d(x)$ and perform pixel shift towards the edge.

4. Interpolate the image from the warped grid to the old uniform grid.

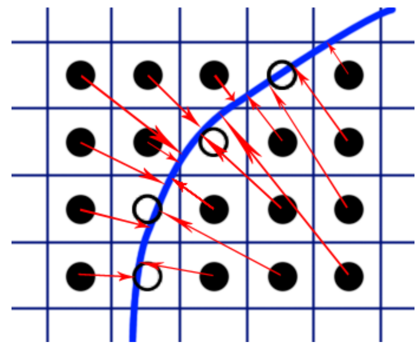


Fig. 2: Displacements for two-dimensional grid warping. Thick blue line represents the exact edge location, white circles represent edge pixels, black circles represent pixels from the edge neighborhood.

The edge map has a great influence on the result of grid warping as only detected edges will be sharpened. We use the result of Canny edge detection [13] as the input of the algorithm. The result of image warping is highly dependent on the parameters of the Canny method (σ and high threshold T_{high}).

2) Multi-edge algorithm: Multi-edge algorithm performs grid warping towards all edges from the neighborhood of the processed pixel using weighted averaging of individual warping vectors for each edge pixel. The weights are proportional to the distance to edge pixels, to gradient magnitude in edge pixels and to the angle between the gradient direction and the line connecting edge pixels and the processed pixel:

$$\vec{d}(P) = \frac{\sum_{Q \in E(P)} w(P,Q)\vec{n}(Q)d((\vec{n}(Q), P-Q))}{\sum_{Q \in E(P)} w(P,Q)},$$

where $\vec{n}(Q) = \frac{\vec{g}(Q)}{|\vec{g}(Q)|}$ is the unit vector corresponding to edge profile (gradient) direction, $(\vec{n}(Q), P-Q)$ is the projection of the vector $\overrightarrow{PQ}$ onto the one-dimensional edge profile, $E(P)$ is the set of edge pixels Q in the neighborhood of the processed pixel P and $w(P,Q)$ is weight coefficient. The size of the neighborhood is chosen in accordance to the support of the displacement function $d(x)$.

The weight coefficient $w(P,Q)$ is defined as

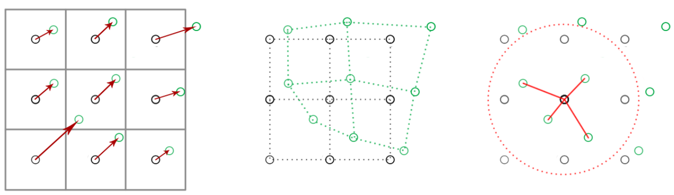
$$w(P,Q) = |\vec{g}(Q)| \exp\left(-\frac{|P-Q|^2 - (P-Q, \vec{n}(Q))^2}{2\sigma_w^2}\right),$$

where the value $|P-Q|^2 - (P-Q, \vec{n}(Q))^2$ is the squared rejection of the vector $P-Q$ onto edge profile direction. The value σ_w is chosen proportional to the blur level. For instance, for Gaussian blur with parameter σ, we use $\sigma_w = 2.5\sigma$.

3) Interpolation: The idea of interpolation from the warped grid to the uniform grid is as follows: the value of pixel P is computed as a weighted sum of intensities of all points on the warped grid in the neighborhood of the pixel (see Fig. 3c): for a given radius ρ and all neighboring points $N(P) = \{Q : |Q-P| \leq \rho\}$ the value of a warped image I_w at P is computed as

$$I_w(P) = \frac{\sum_{Q \in N(P)} \frac{1}{|P-Q|} I(x_k, y_k)}{\sum_{Q \in N(P)} \frac{1}{|P-Q|}}.$$

We use the interpolation radius $\rho = 1.5$. Its size is determined by the maximal distance between pixels after grid warping. The interpolation step introduces small blur effect but its influence on the sharpness improvement is very small.



(a) displacements (b) warped grid (c) interpolation

Fig. 3: Interpolation after grid warping [14].

III. Corner processing

A. The problem

Sharp corners of image edges are represented by high-frequency components. When the image is blurred, the high-frequency information is lost, and the edge corners become rounded (see Fig. 4). Restoring the missing high-frequency information is a challenging problem, and the edge corners often remain rounded.

We suggest an approach based on calculating the edge curvature and adjusting the warping vectors in order to restore the corner sharpness.

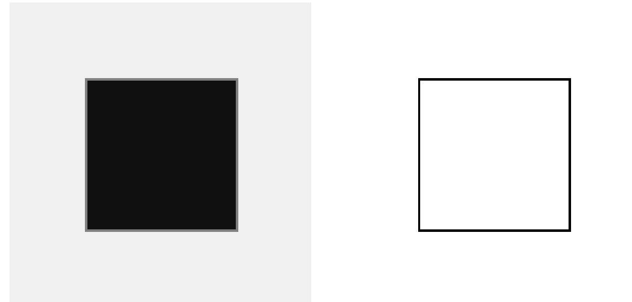
B. Curvature

We calculate edge curvature using curvature analysis of the vector field derived from the image gradient field. For each edge pixel, the direction of the gradient is orthogonal to the direction of the edge. We consider a vector field $\vec{V} = (u, v)$ constructed from normalized image gradients rotated by 90 degrees clockwise:

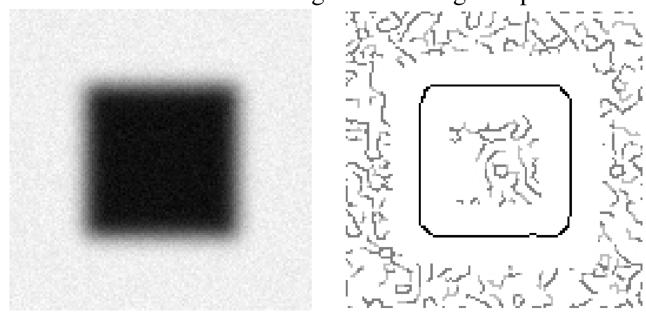
$$u(x,y) = \frac{I_y}{\|\nabla I\|}, \quad v(x,y) = -\frac{I_x}{\|\nabla I\|}.$$

In this case, the edges become *tangent curves* of field $\vec{V}$.

A curve L is called tangent curve of the vector field $\vec{V}$ if the following condition is satisfied: For all points $P \in L$, the tangent vector of the curve in the point P has the same direction as the vector $\vec{V}(P)$.



Ground truth image and its edge map



Blurred and noisy image

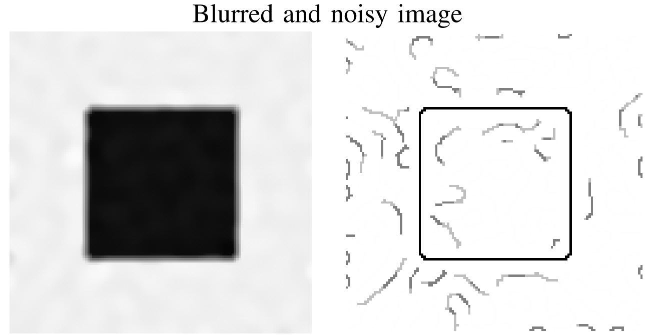
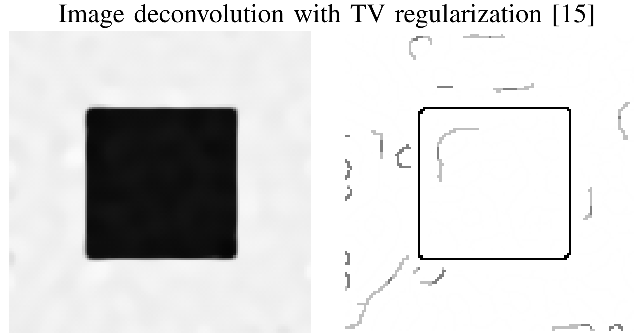


Image deconvolution with TV regularization [15]



Deconvolution + warping post-processing [12]

Fig. 4: Edge contour deformation after image deblurring.

A curvature of a plane curve $L(t) = (x(t), y(t))$, with constant velocity condition

$$(x'(t)^2 + y'(t)^2) \equiv 1 \tag{3}$$

is calculated as

$$\kappa(t) = \|L''(t)\|.$$

The unit normal vector

$$N(t) = \frac{L''(t)}{\|L''(t)\|}$$

describes the rotation direction.

In order to calculate the curvature of the edge at the point $P = (x_0, y_0)$, we do not need to find the entire curve $L(x(t), y(t))$ passing through the point P at some $t_0 : x_0 = x(t_0), y_0 = y(t_0)$. We need to calculate only the second order derivative $L''(t_0)$.

The first order derivative of L equals to the vector $V(x_0, y_0)$ by definition and constraints:

$$L'(t_0) = (x'(t_0), y'(t_0)) = \vec{V}(x_0, y_0).$$

The second order derivative is calculated using the chain rule:

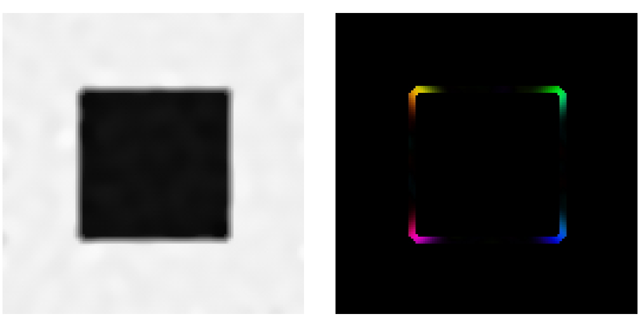
$$L''(t_0) = (x''(t_0), y''(t_0)) = \frac{d\vec{V}(x_0, y_0)}{dt} = \\ = \vec{V_x}u(x_0, y_0) + \vec{V_y}v(x_0, y_0).$$

To improve the accuracy of curvature calculation, we express $\vec{V_x}$ and $\vec{V_y}$ analytically through partial derivatives of the image I and calculate them as convolution with derivatives of a Gaussian filter:

$$G[\sigma](x, y) = \frac{1}{2\pi\sigma^2} \exp\left(-\frac{x^2 + y^2}{2\sigma^2}\right),$$
$$I_x = I * G'_x[\sigma],$$
$$I_y = I * G'_y[\sigma],$$
$$I_{xx} = I * G''_{xx}[\sigma],$$
$$\text{and so on}$$

The parameter σ controls the smoothness of the obtained curvature. Low values of σ produce more accurate results but are sensitive to the noise, while high values σ reduce the influence of the noise at the cost of less accurate results.

Fig. 5 shows an example of curvature and rotation direction calculation.



Input image The result

Fig. 5: An example of curvature and rotation direction calculation. Color (hue) corresponds to the direction of the rotation vector while the intensity is proportional to the curvature value.

C. Warping vectors adjustment

After calculating the curvature map, we adjust the displacement function for each edge point P profile according to rotation direction and curvature value: we perform additional shift of the edge centerline by some value s depending on the curvature value. In order to make the result smooth, we use the Gaussian function:

$$d*(x)[a] = d(x*(x))[a],$$
$$x*(x) = x + s\exp(-\frac{x^2}{8a^2}).$$

Although the research of choosing the optimal coefficient s has not been completed yet, we suggest the following restrictions to the value of s:

1. The rotation direction vector is projected to the edge profile in order to choose the right direction of the edge position adjustment.

2. The coefficient s depends of the curvature value and edge blur level:

$$s = s(\kappa, a).$$

3. The adjustment should be made only when curvature is high:

$$\kappa > f(a) \Rightarrow s = 0.$$

Here $f(a)$ is a threshold function depending on the edge blur level a. The function is calculated experimentally.

4. The maximal shift should be limited according to the blur level:

$$|s| < a/2.$$

D. Discussion

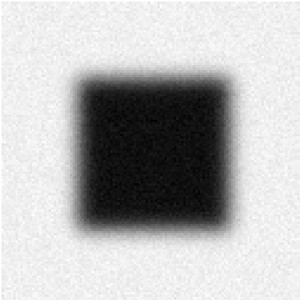
We have applied the proposed algorithm to photographic images from TID database [16] with modeled blur and noise artifact. We have compared the results of the described above original grid warping algorithm [12] with its proposed enhancement. Warping is applied as a post-processing algorithm after regularization-based image deconvolution with Total Variation stabilizer [15].

The result of applying the proposed algorithm are shown in Fig. 6 and Fig. 7. Compared to the original grid warping algorithm, the improved version makes the image corners less rounded. This effect improves the perceived sharpness.

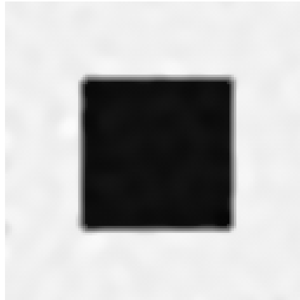
The objective PSNR and SSIM values are the same for both algorithms. This is caused by the fact that only few corner pixels are processes and its contribution to overall image metrics values is negligible.

IV. Conclusion

A modification of grid warping based image sharpening algorithm has been proposed. Grid warping approach is able to increase the edge sharpness without introducing ringing artifact or noise amplification. It has been shown that the proposed adjustment of the warping vectors using the analysis of edge curvature can further improve the perceived sharpness of edge corners.



Blurry and noisy input image

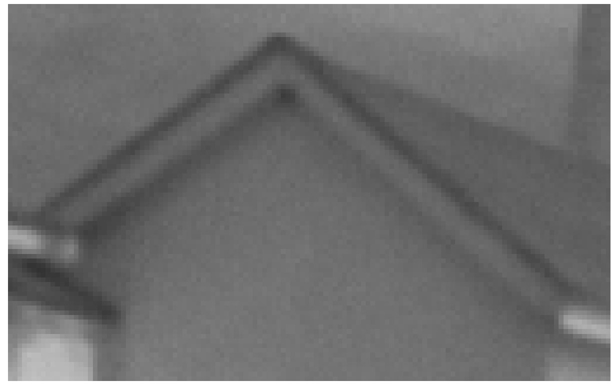


Deconvolution [15]

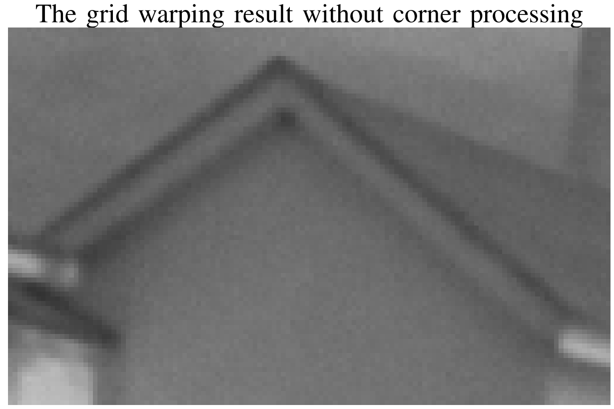
Deconvolution + warping without corner awareness [12]

Deconvolution + proposed method

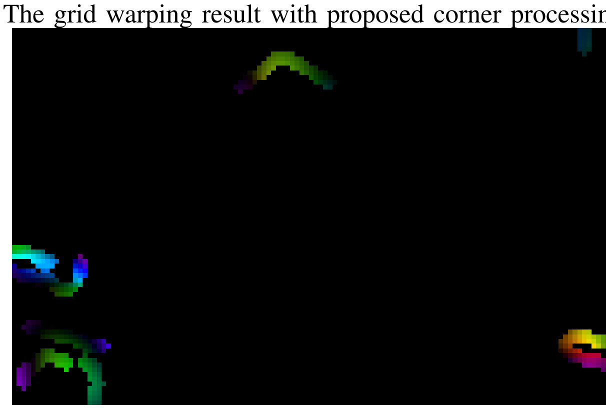
Fig. 6: The result of edge corner sharpening by the proposed algorithm for the synthetic image.



The grid warping result without corner processing



The grid warping result with proposed corner processing



The curvature map

Fig. 7: The result of edge corner sharpening by the proposed algorithm for the image from TID database.

References

[1] J. Oliveira, J. M. Bioucas-Dia, and M. Figueiredo. Adaptive total variation image deblurring: A majorization-minimization approach. *Signal Process.*, 89:1683–1693, 2009.

[2] M.S.C. Almeida and M.A.T. Figueiredo. Parameter estimation for blind and non-blind deblurring using residual whiteness measures. *IEEE Trans. Image Processing*, 22: 2751–2763, 2013.

[3] S. D. Babacan, R. Molina, and A. K. Katsaggelos. Variational bayesian blind deconvolution using a total variation prior. *IEEE Trans. Image Process.*, 18:12–26, 2009.

[4] N. Arad and C. Gotsman. Enhancement by image-dependent warping. *IEEE Trans. Image Proc.*, 8:1063–1074, 1999.

[5] J. Prades-Nebot et al. Image enhancement using warping technique. *IEEE Electronics Letters*, 39:32–33, 2003.

[6] J.G.M. Schavemaker, M.J.T. Reinders, J.J. Gerbrands, and E. Backer. Image sharpening by morphological filtering. *Pattern Recognition*, 33(6):997—1012, 2000.

[7] Stanley Osher and Leonid I. Rudin. Feature-oriented image enhancement using shock filters. *SIAM Journal on Numerical Analysis*, 27(4):919–940, 1999.

[8] Guy Gilboa, Nir A. Sochen, and Yehoshua Y. Zeevi. Regularized shock filters and complex diffusion. *Lecture Notes in Computer Science*, 2350:399–413, 2002.

[9] Joachim Weickert. Coherence-enhancing shock filters. *Lecture Notes in Computer Science*, 2781:1–8, 2003.

[10] Andrey Krylov, Alexandra Nasonova, and Andrey Nasonov. Image warping as an image enhancement post-processing tool. *OGRW2014*, pages 132–135, 2015.

[11] A. Nasonova and A. Krylov. Deblurred images post-processing by Poisson warping. *IEEE Signal Processing Letters*, 22(4):417–420, 2015.

[12] Andrey Krylov, Andrey Nasonov, and Yakov Pchelintsev. Single parameter post-processing method for image deblurring. *Seventh International Conference on Image Processing Theory, Tools and Applications (IPTA2017)*, pages 1–6, 2017.

[13] J. Canny. A computational approach to edge detection. *IEEE Trans. Pattern Analysis and Machine Intelligence*, 8:679–698, 1986.

[14] A. Krylov, A. Nasonova, and A. Nasonov. Grid warping for image sharpening using one-dimensional approach. In *Proceedings of International Conference on Signal Processing (ICSP2014)*, pages 672–677, Hangzhou, China, 2014. IEEE.

[15] Leonid I Rudin, Stanley Osher, and Emad Fatemi. Nonlinear total variation based noise removal algorithms. *Physica D: nonlinear phenomena*, 60(1-4):259–268, 1992.

[16] N. Ponomarenko, L. Jin, O. Ieremeiev, V. Lukin, K. Egiazarian, J. Astola, B. Vozel, K. Chehdi, M. Carli, F. Battisti, and C.-C. Jay Kuo. Image database TID2013: Peculiarities, results and perspectives. *Signal Processing: Image Communication*, 30:57–77, 2015.