GRID WARPING IN TOTAL VARIATION IMAGE ENHANCEMENT METHODS

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ABSTRACT

An approach of including warping algorithms into total variation image enhancement methods is suggested. The idea of warping is to make edges sharper using pixel grid transform so that the pixels near edges move closer to the edges. The advantage of using warping approach is that the change of the total variation value is small, so it can be used to improve the results of total variation image enhancement methods like deblurring, deringing and resampling.

Index Terms— Image grid warping, total variation, image enhancement

1. INTRODUCTION

Algorithms based on Total Variation (TV) minimization have become widely used in image enhancement due to the peculiar features that cannot be realized with smooth regularization [1, 2]. Classic TV method for image denoising was introduced in [3]. Now TV methods are used for a large variety of image restoration areas including image denoising [4], deblurring [5], deringing [6], resampling and super-resolution [7], inpainting [8] and other areas.

Typically the TV methods are posed using unconstraint minimization of the regularization functional

$$f(z) = D(z_0, z) + \lambda TV(z),$$

or constrained minimization on the set of images with bounded TV

$$\min_{TV(z)\leq C} D(z_0, z).$$

The term $D(z_0, z)$ is the data-fitting term, the regularization term $TV(z)$ is the total variation functional

$$TV(z) = \int |\nabla z| dz,$$

and λ is the regularization parameter.

Usually the data-fitting term looks as

$$D(z_0, z) = \|Az - z_0\|_Z,$$

where operator A corresponds to the posed problem: for noise reduction and ringing suppression operator A is usually the identity operator, for deblurring the blur kernel is used as operator A.

Typical effect of image total variation minimization is the following: it flattens the areas with small intensity changes corresponding to noise and ringing while keeping strong edges intact. If A is the identity operator, the edge sharpness is not changed. If A is the blur kernel, then the edges become a bit sharper.

We propose a way to improve the result of total variation image enhancement using grid warping that has almost no effect on the total variation. The idea of grid warping is to make edges sharper using pixel grid transform so that the pixels near edges move closer to the edges.

The existing warping approaches [9, 10] were not used in conjunction with TV image enhancement methods. The method [9] does not limit the warping effect to the areas near the edges and can produce unwanted image distortions. The method [10] works independently in horizontal and vertical directions and can introduce local changes in edge directions.

The proposed image warping method is more suitable to be used simultaneously with TV algorithms.

2. GRID WARPING

2.1. 1D edge sharpening

We assume that a blurred edge can be approximated by a step edge function $H(x)$ smoothed with a Gaussian kernel:

$$E_\sigma(x) = [H * G_\sigma](x), \qquad H(x) = \begin{cases} 1, & x \geq 0, \\ 0, & x < 0, \end{cases}$$

where

$$G_\sigma(x) = \frac{1}{\sqrt{2\pi}\sigma} e^{-\frac{x^2}{2\sigma^2}}.$$

The 1D grid warping looks as follows. Let $\widetilde{x} = \widetilde{x}(x)$ be the new coordinate of the point x. Consider one-dimensional edge profile $E(x)$ centered at the zero point and profile of the warped edge $E(\widetilde{x})$. For edge sharpening, we use the transform:

$$\overline{x} = x + AG'_\sigma(x) \tag{1}$$

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where constant A is equal to

$$A = \frac{1}{G'_\sigma(0)}.$$

2.2. 2D image warping

The following algorithm is used:

1. Perform Canny edge detection [11] with zero thresholds and σ corresponding to edge model that is given or estimated by the edge width estimation algorithm proposed in [12, 13].

2. For every pixel take all edges from its neighborhood, take directions normal to the edges, find pixel shifts (1) and compute the average shift vector with weights proportional to edge gradient values.

3. Interpolate the image from the warped grid to the old uniform grid. The pixels $I_s(i, j)$ are interpolated by a weighted averaging of neighbor pixels in a given radius r of the transformed grid $I(x_k, y_k)$:

$$I_s(i, j) = \frac{\sum d_k I(x_k, y_k)}{\sum d_k},$$

where $d_k = 1/\sqrt{(i - x_k)^2 + (j - y_k)^2}$. In this work we use $r = 1.5$.

2.3. Effects of warping on the total variation

The most important effect of warping on the total variation is that it does not change the TV value of 1D function in the continuous case. This fact comes from the definition of total variation [14]. In 1D, the TV is defined by the formula

$$TV(E) = \int |E'(x)| dx.$$

The TV of the warped edge is equal to

$$TV(\widetilde{E}) = \int |(E(\widetilde{x})'| dx = \int |E'(\widetilde{x})\widetilde{x}'| dx =$$
$$= \int |E'(\widetilde{x})| \widetilde{x}' dx = \int |E'(\widetilde{x})| d\widetilde{x} = TV(E).$$

In real applications, image total variation can slightly change after warping (usually decrease). This is caused by change of the local length of level set lines connected with the TV value [14] and smoothing at the interpolation step. Nevertheless the relative TV change is small. We will check the change experimentally in the Results section.

3. APPLICATION OF THE GRID WARPING FOR TV IMAGE ENHANCEMENT

We apply grid warping as a postprocessing algorithm to TV image enhancement methods. To demonstrate the effectiveness of the proposed method, we took 29 reference images from the LIVE database [15] and created downsampled images and images with blur, noise and ringing effect. Then we applied TV based image enhancement algorithms to the degraded images and performed edge sharpening using the proposed method. Average PSNR and TV values before and after image warping were calculated and compared.

3.1. TV image enhancement methods

3.1.1. *Image deblurring*

Blurred images were generated from reference images using Gaussian filter with radius $\sigma = 3$ and Gaussian noise with standard deviation 5. We consider the following algorithms of TV image deblurring:

1. Projection of the image sharpened by unsharp masking with high α onto the set of images with bounded TV [16]:

$$\begin{aligned} z_\alpha &= Hz_0 + \alpha(z_0 - Hz_0), \\ z_R &= \arg \min_{TV(z) \le TV(z_0)} \|z_\alpha - z\|_2, \end{aligned} \tag{2}$$

where H is the Gaussian filter with the same parameter σ as was used to generate blurred image. We use $\alpha = 5$.

2. Splitting the image into high-frequency and low-frequency parts and deblurring only low frequency part using the previous algorithm [16]:

$$\begin{aligned} z_L &= Hz_0 \\ z_H &= z_0 - z_L \\ z_\alpha &= Hz_L + \alpha(z_L - Hz_L) \\ z_R &= z_H + \arg \min_{TV(z) \le TV(z_L)} \|z_\alpha - z\|_2. \end{aligned} \tag{3}$$

3.1.2. *Image deringing*

Ringing effect was modeled by ideal low-pass filtering with the cut-off frequency 0.125 Hz that produces ringing oscillations with the width 4. Gaussian noise with standard deviation 5 was added to the images.

The connection between ringing effect and TV is shown in [14]. When ringing effect appears, the TV strongly increases. To suppress ringing effect, we project the image onto the set of images with bounded total variation:

$$z_\beta = \arg \min_{TV(z) \le \beta TV(z_0)} \|z - z_0\|_2. \tag{4}$$

Parameter $0 \le \beta \le 1$ controls the ringing suppression level. The less is β, the stronger ringing suppression is. The value $\beta = 1$ means no ringing suppression. We use $\beta = 0.7$ which maximizes the PSNR of images after deringing for the LIVE database.

Method	No warping		After warping	
	PSNR	TV	PSNR	TV
Source image	—	15.58		
Blurred and noisy	21.61	9.59		
Deblurring (2)	21.77	10.24	21.91	9.80
Deblurring (3)	21.79	10.14	21.88	9.16
Noisy with strong ringing	23.30	11.94		
Deringing (4)	23.51	8.63	23.61	8.32
Resampling (5)	23.35	4.03	23.63	4.27

Fig. 1. Average PSNR and TV values of image enhancement algorithms without and with applying the proposed image warping method for the images from the LIVE database [15]

3.1.3. Image resampling

The TV based image resampling algorithm does not require postprocessing to improve image sharpness if the input low-resolution image is sharp. But if the low-resolution image is blurred then the high-resolution image is blurred too and edge sharpening will show effective results. The images were downsampled with the factor 2, blurred with Gaussian filter with $\sigma_D = 1.5$ and then resampled using the TV based regularization method [17]

$$z_\lambda = \arg\min\left(\|DH_0 z - z_0\|_2 + \lambda TV(z)\right) \qquad (5)$$

where H_0 is Gaussian filter with $\sigma_0 = 0.4$ used as anti-aliasing filter and D is the decimation operator, $\lambda = 0.03$ is the regularization parameter. TV stabilizer is used here as the a priory information to reconstruct pixels in high-resolution image.

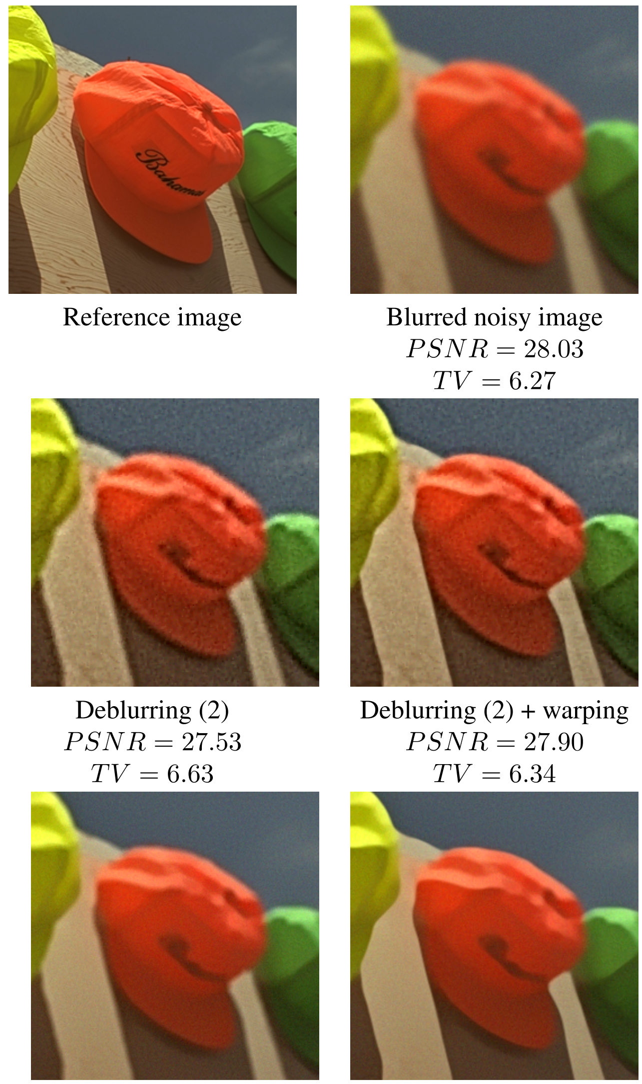
3.2. Results

The overall results of applying the proposed method after TV based image restoration algorithms are shown in Table 1. Performing grid warping after image enhancement algorithms increases both PSNR value and visual quality in all cases.

The results of applying the grid warping algorithm for deblurring of blurred and noisy image are shown in Fig. 2. It can be seen that applying grid warping after image deblurring increases edge sharpness and overall image quality. Moreover the edges become not only sharper but straighter too. Increasing the parameter α in (2) and (3) instead of using image warping to sharp the edges will degrade image quality due to noise amplification.

The results of applying image warping for the TV deringing are shown in in Fig. 3 and the results for image resampling are shown in Fig. 4.

To show the effectiveness of the proposed algorithm for real images, we took outfocued noisy image from a camera and applied deblurring (3) with warping. The results are shown in Fig. 5.



Reference image

Blurred noisy image
$PSNR = 28.03$
$TV = 6.27$

Deblurring (2)
$PSNR = 27.53$
$TV = 6.63$

Deblurring (2) + warping
$PSNR = 27.90$
$TV = 6.34$

Deblurring (3)
$PSNR = 28.54$
$TV = 6.61$

Deblurring (3) + warping
$PSNR = 29.10$
$TV = 5.94$

Fig. 2. The result of applying image warping for image deblurring

Reference image

Ringing + noise
$PSNR = 27.33$
$TV = 8.03$

Deringing
$PSNR = 27.84$
$TV = 2.65$

Deringing + warping
$PSNR = 27.03$
$TV = 2.55$

Fig. 3. The result of applying image warping for image deringing

4. CONCLUSION

The warping algorithm for edge sharpening has been presented. The future work will include pure 2D warping algorithms.

Application of the proposed method as a post-processing algorithm to TV based image enhancement algorithms has shown its effectiveness. Moreover, the warping and TV ideas are complementary and there can be other effective variants of simultaneous use of TV methods and warping algorithm.

5. REFERENCES

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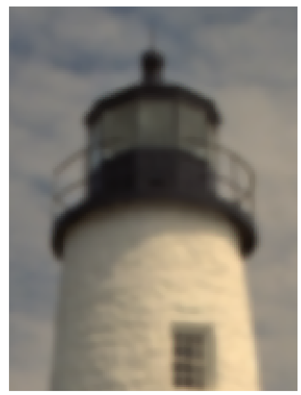
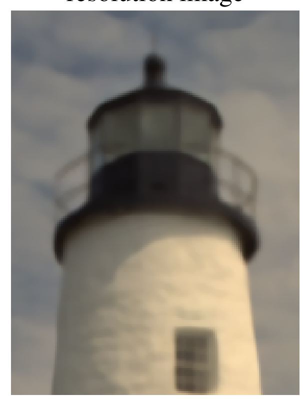
Reference image

Blurred low resolution image



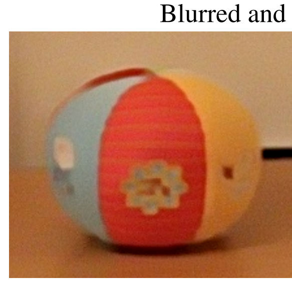
Resampling
$PSNR = 23.67$
$TV = 3.07$



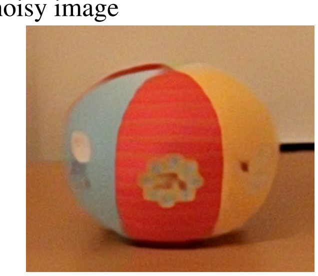
Resampling + warping
$PSNR = 23.75$
$TV = 3.07$

Fig. 4. The result of applying image warping for image resampling

Blurred and noisy image



Deblurring (3)



Deblurring (3) + warping

Fig. 5. The result of applying image warping for image deblurring

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