# Segmentation using Level Set Methods

## Region Based Active Contours

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# Active Contours (Snakes)

- Active contour model (snakes) can be written as:

$$E_{AC}(\mathcal{C}) = \int_0^1 \alpha|\mathcal{C}'(q)|^2 + \beta|\mathcal{C}''(q)|^2 dq + \int_0^1 P(\mathcal{C}(q))dq,$$

  where $P$ is the potential field.
- The deformable contour (snake) is a mapping:

$$\mathcal{C}(q) : [0,1] \to \mathbb{R}^2, \quad q \mapsto \mathcal{C}(q) = (x(q), y(q))^T.$$

- Potential $P$ can be:
  - Edges: $P_{edge}(\mathcal{C}(q)) = -|\nabla(G_\sigma * f(\mathcal{C}(q)))|^2$
  - Lines (high intensity): $P_{line}(\mathcal{C}(q)) = -G_\sigma * f(\mathcal{C}(q))$
  - Combination: $P(\mathcal{C}(q)) = -w_{line}P_{line} - w_{edge}P_{edge}$
  - Often $P(\mathcal{C}(q)) = g(|\nabla G_\sigma * I(\mathcal{C}(q))|)$ where $g : [0,\infty) \to \mathbb{R}^+$ is a strictly decreasing function: $g(r) \to 0$ as $r \to \infty$.

# Active Contours: Evolution equation

- The snake evolution equation:

$$\frac{\partial \mathcal{C}}{\partial t} = \alpha \mathcal{C}''(s) - \beta \mathcal{C}''''(s) - k_1 \mathbf{n}(s) - k_2 \frac{\nabla P(\mathcal{C}(s))}{|\nabla P(\mathcal{C}(s))|}$$

  where

$$P(\mathcal{C}(s)) = -w_{line}(G_\sigma * f(\mathcal{C}(s))) + w_{edge}|\nabla(G_\sigma * f(\mathcal{C}(s)))|^2$$

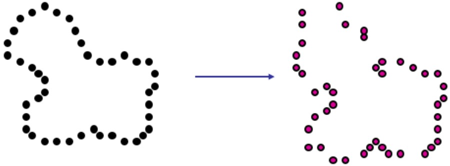
- In matrix form:

$$(I + \tau A)X^{t+1} = X^t + \tau F(X^t)$$

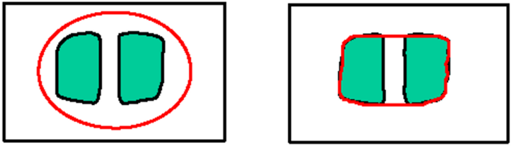
- Parameter choice (better but not obligatory):
  - $\alpha$ controls elasticity
  - $\beta$ controls stiffness
  - $k_1$ sign controls inflate or deflate
  - $|\tau k_1| < |\tau k_2| < 1$
  - $\tau$ controls the snake speed

# Problems with Parametric Curves

- Reparametrization needed (hard with surfaces in 3D)



- Cannot handle topological changes



- Hard to extend to 3D

$$\gamma\frac{\partial \mathbf{v}}{\partial t} - \frac{\partial}{\partial s}\left(w_{10}\frac{\partial \mathbf{v}}{\partial s}\right) - \frac{\partial}{\partial r}\left(w_{01}\frac{\partial \mathbf{v}}{\partial r}\right) + 2\frac{\partial^2}{\partial s \partial r}\left(w_{11}\frac{\partial^2 \mathbf{v}}{\partial s \partial r}\right)$$
$$+\frac{\partial^2}{\partial s^2}\left(w_{20}\frac{\partial^2 \mathbf{v}}{\partial s^2}\right) + \frac{\partial^2}{\partial r^2}\left(w_{02}\frac{\partial^2 \mathbf{v}}{\partial r^2}\right) + \nabla P(\mathbf{v}(s,r)) = 0,$$

# Geodesic Active Contours

- Caselles et.al. 1995, Kichenassamy et al. 1995
- Curve with minimal geodesic length is searched.

$$E_{GAC}(\mathcal{C}) = \int_0^{L(\mathcal{C})} g(|\nabla G_\sigma * I(\mathcal{C}(s))|) ds =$$
$$= \int_0^1 g(|\nabla G_\sigma * I(\mathcal{C}(q))|) |\mathcal{C}'(q)| dq$$

where $g : [0, \infty) \to \mathbb{R}^+$ is a strictly decreasing function such that $g(r) \to 0$ as $r \to \infty$, e.g.,

$$g(|\nabla G_\sigma * I(x,y)|) = \frac{1}{1 + |\nabla G_\sigma * I(x,y)|}$$

- The curve is attracted by image edges, where the weight $g(|\nabla G_\sigma * I(C(q))|)$ is small.
- Energy functional is not convex and therefore there are several local minima.

# Geodesic Active Contours: Evolution Equation

- Evolution equation (i.e. Euler-Lagrange equation with $\frac{\partial \mathcal{C}}{\partial t}$ on the left side) for geodesic active contours is

$$\frac{\partial \mathcal{C}}{\partial t} = (g\kappa - (\nabla g \cdot \mathbf{n}))\,\mathbf{n}$$

  where **n** is curve normal (vector) and $\kappa$ is curvature (scalar)
- This equation can be rewritten in level-set framework.
- The curve is embeded in SDF $u$ (with evolution speed $\nu = g\kappa - \nabla g \cdot n$).
  The evolution equation becomes

$$\frac{\partial u}{\partial t} = g\kappa|\nabla u| - \nabla g \cdot \nabla u$$

# Level Sets Idea

- Curve $\mathcal{C}$ is represented implicitly as a zero level set of a higher-dimensional function $u : \mathbb{R}^2 \to \mathbb{R}$.

  $$\mathcal{C} = \{(x, y) : u(x, y) = 0\}$$

  In level set formulation, the curve evolution according to

  $$\frac{\partial \mathcal{C}}{\partial t} = \beta \boldsymbol{n}$$

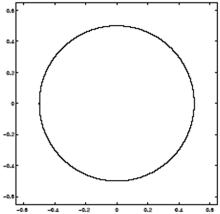
  leads to the evolution of embedding function $u$ according to

  $$\frac{\partial u}{\partial t} + \beta |\nabla u| = 0$$

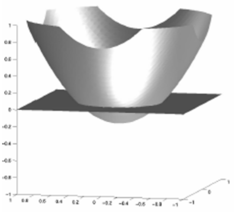
  where $\boldsymbol{n}$ is curve normal and $\beta$ is the evolution speed (scalar).

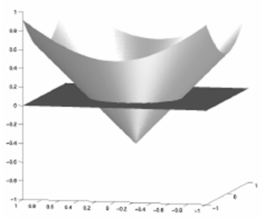
# Level Sets Idea

- Curve



- Different embedding functions $u(x, y)$

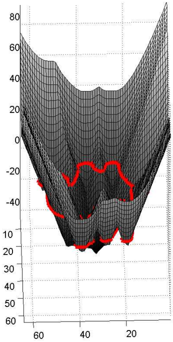




# Signed Distance Function (SDF): Example

- For $C = \partial\Omega$, SDF $d$ defined by:
$$d(\mathbf{x}) = \begin{cases} -\min_{\mathbf{y}\in C} |\mathbf{x}-\mathbf{y}| & \text{if } \mathbf{x} \in \Omega^- \\ +\min_{\mathbf{y}\in C} |\mathbf{x}-\mathbf{y}| & \text{if } \mathbf{x} \in \Omega^+ \cup C \end{cases}$$
- Normal $n = \nabla d\,,\ |\nabla d| = 1$
- Curvature $\kappa = \nabla \cdot \nabla d = \nabla^2 d = \Delta d$

digital shape




SDF $d(\mathbf{x})$

# Geodesic Active Contours: Evolution Equation

- The evolution equation

$$\frac{\partial u}{\partial t} = g\kappa|\nabla u| - \nabla g \cdot \nabla u$$

  contains curvature motion and velocity field motion.
- It is solved using level set methods
- We can add normal direction motion (baloon force)

$$\frac{\partial u}{\partial t} = (c + \kappa)g|\nabla u| - \nabla g \cdot \nabla u$$

# Geodesic Active Contours: Iterative Scheme

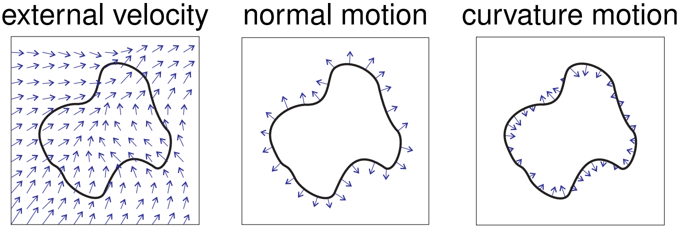
- The equation

$$u_t = (c + \epsilon\kappa)g|\nabla u| + \beta\nabla g \cdot \nabla u$$

  consists of three types of motion we have discussed before:
  - normal direction motion with speed $cg(|\nabla G_\sigma * I(x,y)|)$
  - curvature motion multiplied by a factor $\epsilon g(|\nabla G_\sigma * I(x,y)|)$
  - external velocity field motion given by $\beta\nabla g$.
- Therefore, we have

$$u_{ij}^{k+1} = u_{ij}^{k} + \tau \cdot [\text{Normal}(cg) + \text{Curvature}(\epsilon g) - \text{Velocity}(\beta\nabla g)].$$

# Three types of motion



# All Types of Motion Together

The general equation

$$u_t = a|\nabla u| - \epsilon\kappa|\nabla u| + \beta V \cdot \nabla u$$

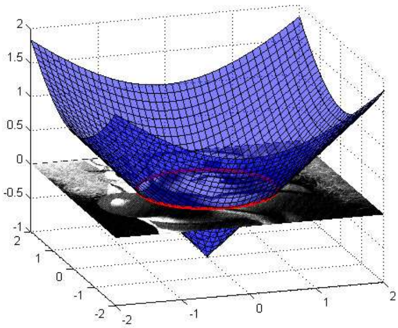
is discretized with the numerical scheme

$$u_{ij}^{n+1} = u_{ij}^n + \tau \left[ \begin{array}{c} [\max(cg_{ij}, 0)\nabla^+ + \min(cg_{ij}, 0)\nabla^-] + \\ +[\epsilon g_{ij} K_{ij}^n \sqrt{(D_{ij}^{0x})^2 + (D_{ij}^{0y})^2}] + \\ \left[ \begin{array}{c} \max(w_{ij}^n, 0) D_{ij}^{-x} + \min(w_{ij}^n, 0) D_{ij}^{+x} \\ + \max(v_{ij}^n, 0) D_{ij}^{-y} + \min(v_{ij}^n, 0) D_{ij}^{+y} \end{array} \right] \end{array} \right],$$

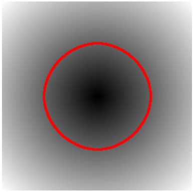
Note: $w_{ij} = {g'}_x(ih, jh)$ and $v_{ij} = {g'}_y(ih, jh)$

Stability condition: The most restrictive (curvature) term forces $\tau = \mathcal{O}(h^2)$

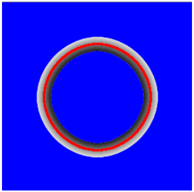
# Level Set Methods: Narrow Band




Whole domain

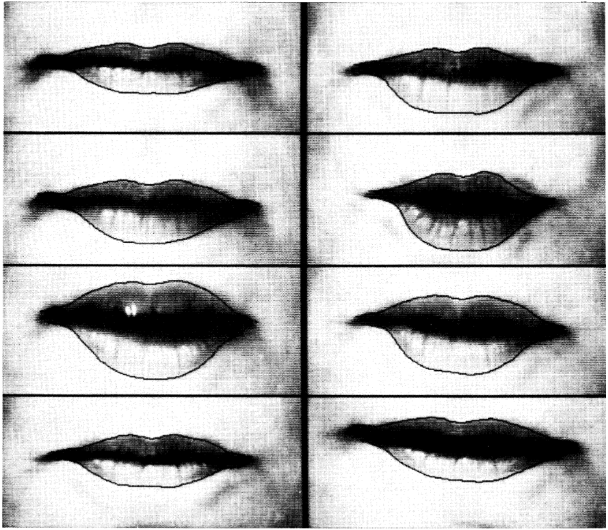


Narrow band



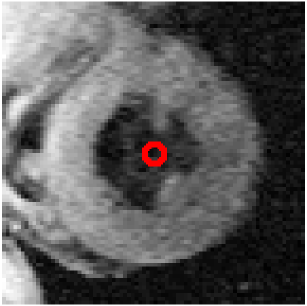
- We can compute the evolution in a narrow band around the zero level set only!

# Active Contours: Example

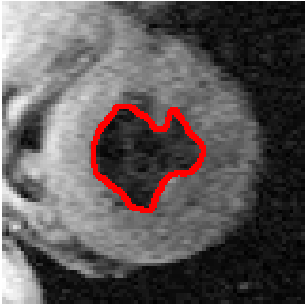


Source: [Kass et. al. 1987]

# Geodesic Active Contours: Example



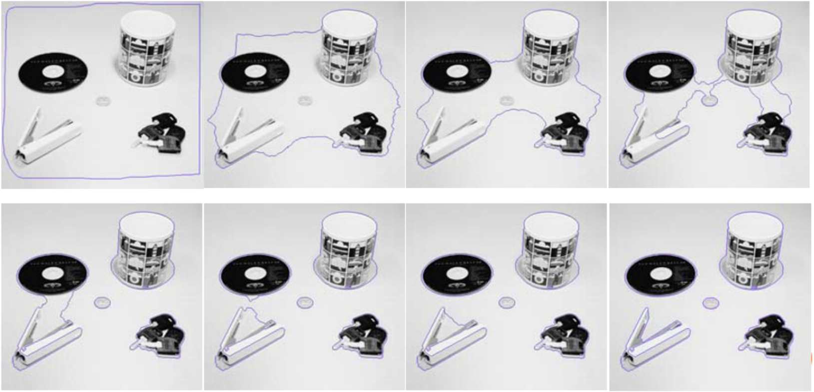
Initial contour



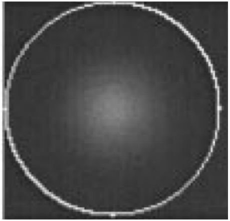
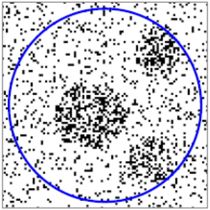
Result

$\tau = 0.25, c = 1, \epsilon = 0.5,$
$\beta = 0.5, p = 2, \sigma = 2.0$

# Geodesic Active Contours: Example



# What if no / weak / noisy edges?

- Region interior was not considered in previously discussed active contours!
- No region homogeneity was required (image structures under the curve are important for solution only).

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# Mumford-Shah Functional

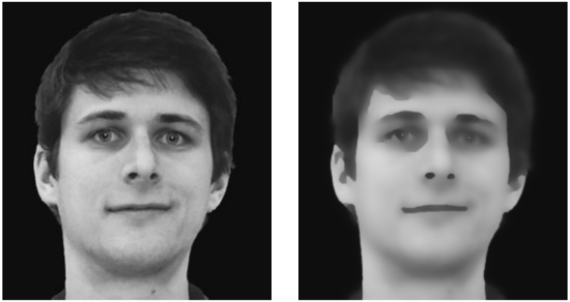
- Segmentation $(u, \mathcal{C})$ of an image $f : \Omega \to \mathbb{R}$ is defined as the minimizer of

$$E_{MS}(u, \mathcal{C}) = \lambda \int_{\Omega} (u - f)^2 dx + \beta \int_{\Omega \setminus \mathcal{C}} |\nabla u|^2 dx + \mu |\mathcal{C}|$$

  where $u(x)$ is smoothed version of $f(x)$, and $\mathcal{C}$ is an edge set curve where $u$ is allowed to be discontinuous
- First term penalises deviations from original image $f$
- Second term penalises variations within each segment
- Third term penalises the edge length $|\mathcal{C}|$
- Mathematically very difficult:
  - We are looking for edges $\mathcal{C}$ and image $u$
  - Non-convex
  - Complicated and computationally expensive
- No unique solution in general.
- Several simplifications of functional $E_{MS}$ were proposed

# Mumford-Shah Functional: Example

$f$ | Mumford–Shah piecewise-smooth approximation



From [P. Getreuer, Chan–Vese Segmentation, IPOL Journal, 2012]

# Mumford-Shah Functional

- Piecewise constant formulation
- Segmentation $(u, \mathcal{C})$ of an image $f : \Omega \to \mathbb{R}$ is defined as the minimiser of

$$E_{MS1}(u, \mathcal{C}) = \int_{\Omega} (u - f)^2 dx + \mu |\mathcal{C}|$$

  where $u(x)$ u is required to be constant on each connected component of $\Omega \setminus \mathcal{C}$
- Existence of a solution proved by [Mumford and Shah, 1989] and [Morel and Solimini, 1994]

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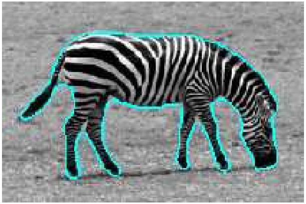
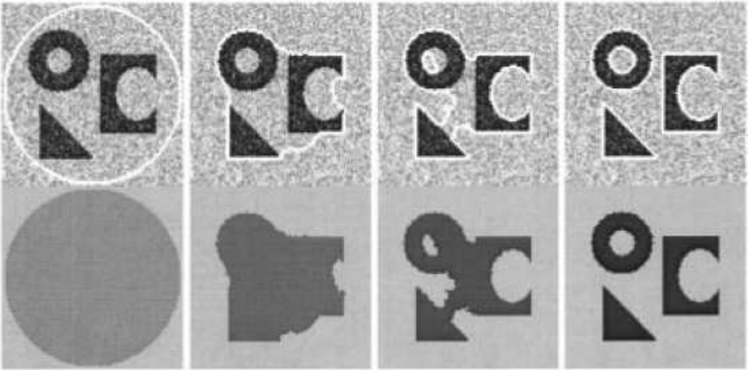
# Active-Contours Without Edges

- Chan and Vese 1998
- **Given**: Image $f : \Omega \to \mathbb{R}$
- **Goal**: Segmentation of $\Omega$ into two regions (possibly disconnected)
- Curve evolution is based on region information (but not on edges)
- Can be extended to segment color and textured images.

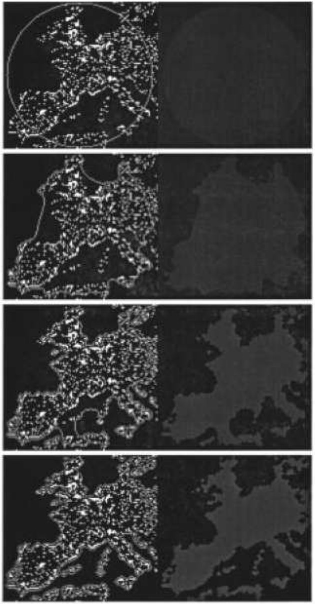


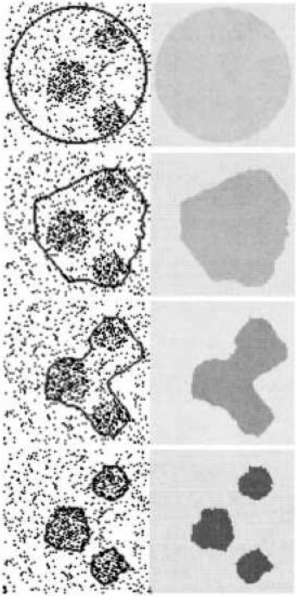
# Active Contours Without Edges: Example 1



Source: Chan, Vese, Active Contours Without Edges, 2001

# Active Contours Without Edges: Example 2





Source: Chan, Vese, Active Contours Without Edges, 2001

# Active-Contours Without Edges: Idea

- Regions are separated by a curve $\mathcal{C}$.
- In each region, a constant grey-value is supposed to approximate the image.
- **Data term** penalises the deviation from the piecewise constant approximation of the input image
- **Regularity term** impose regularity constraints for the curve (requires curve of minimal length).

# Active-Contours Without Edges: Functional

- Let $f(x) : \Omega \to \mathbb{R}$ is the input image.
- Let $\mathcal{C}$ be the boundary between two regions $\Omega_1$ and $\Omega_2$. $\Omega = \Omega_1 \cup \Omega_2 \cup \mathcal{C}$
- Chan-Vese functional is defined

$$E_{CV}(\mathcal{C}, c_1, c_2) = \mu L(\mathcal{C}) + \nu A(\Omega_1) + \\ + \lambda_1 \int_{\Omega_1} |f(x) - c_1|^2 dx + \lambda_2 \int_{\Omega_2} |f(x) - c_2|^2 dx$$

  where $\mu \geq 0, \nu \geq 0, \lambda_1, \lambda_2 \geq 0$ are given fixed parameters (weights), $L(\mathcal{C})$ denotes the length of $\mathcal{C}$, $A(\Omega_1)$ denotes the area inside $\mathcal{C}$, and $c_1$ and $c_2$ are mean intensity values of two distinct regions
- We minimize the functional with respect to $c_1$,$c_2$, and $\mathcal{C}$.

# Equivalence with Mumford-Shah Functional

- Chan-Vese functional:

$$E_{CV}(\mathcal{C}, c_1, c_2) = \mu L(\mathcal{C}) + \nu A(\Omega_1) + \\ + \lambda_1 \int_{\Omega_1} |f(x) - c_1|^2 dx + \lambda_2 \int_{\Omega_2} |f(x) - c_2|^2 dx$$

- Mumford-Shah functional:

$$E_{MS}(u, \mathcal{C}) := \lambda \int_{\Omega} (u - f)^2 dx + \beta \int_{\Omega \setminus K} |\nabla u|^2 dx + \mu |\mathcal{C}|$$

- Key difference:
  - Only two regions (in MS $\lambda = \lambda_1 = \lambda_2$)
  - Area term
  - Piecewise constant approximation $u$ with 2 values

# Chan-Vese Functional: Theory

- For given $\mathcal{C}$, the optimal values $c_1$ and $c_2$ are uniquely determined as the average gray values of $f$ within their respective regions.
- There exist a curve $\mathcal{C}$ of finite length that minimises $E_{CV}(\mathcal{C}) = E_{CV}(\mathcal{C}, c_1(\mathcal{C}), c_2(\mathcal{C}))$ (existence of solution).
- Energy is non-convex, therefore there can exist more then one local minimum.

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# Active-Contours Without Edges: Implementation

- Level set formulation: The curve $\mathcal{C}$ is represented as a zero level set of a continuous function $u : \Omega \to \mathbb{R}$. We get

$$
\begin{aligned}
E_{CV}(\mathcal{C}, c_1, c_2) = E_{CV}(u, c_1, c_2) = \\
&= \mu \int_\Omega |\nabla H(u(x))| dx + \nu \int_\Omega H(u(x)) dx + \\
&+ \lambda_1 \int_\Omega (f(x) - c_1)^2 H(u(x)) dx + \\
&+ \lambda_2 \int_\Omega (f(x) - c_2)^2 (1 - H(u(x))) dx
\end{aligned}
$$

where $H(u)$ is Heaviside function:

$$
H(u) = \begin{cases} 1 & \text{for } u \geq 0 \\ 0 & \text{for } u < 0 \end{cases}
$$

# Active-Contours Without Edges: Implementation

- $H(u)$ is Heaviside function:

$$H(t) = \begin{cases} 1 & \text{for } t \geq 0 \\ 0 & \text{for } t < 0 \end{cases}$$

- $\delta(t) = \frac{d}{dt}H(t)$ is Dirac delta function:

$$\int_{-\infty}^{+\infty} \delta(x) f(x) dx = f(0), \quad \int_{-\infty}^{+\infty} \delta(x) dx = 1$$

- Curve length:

$$L(\mathcal{C}) = \int_\Omega |\nabla H(u(x))| dx = \int_\Omega \delta(u(x)) |\nabla u(x)| dx$$

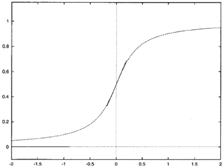
- Constants are average gray values of regions:

$$c_1 = \frac{\int_\Omega f(x) H(u(x)) dx}{\int_\Omega H(u(x)) dx}, \quad c_2 = \frac{\int_\Omega f(x)(1 - H(u(x))) dx}{\int_\Omega (1 - H(u(x))) dx}$$

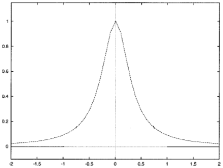
# Active-Contours Without Edges: Implementation

- Regularized version of $H$ is used:

$$H_\epsilon(u) = \frac{1}{2}\left(1 + \frac{2}{\pi} tan^{-1}\left(\frac{u}{\epsilon}\right)\right).$$




- The corresponding $\delta_\epsilon$:



# Evolution Equation

- Minimization of CV functional leads to the following evolution equation:

$$\frac{\partial u}{\partial t} = \delta_\epsilon(u)\left[\mu\nabla\cdot\left(\frac{\nabla u}{|\nabla u|}\right) - \nu - \lambda_1(f - c_1)^2 + \lambda_2(f - c_2)^2\right]$$

  where

$$c_1 = \frac{\int_\Omega f(x)H(u(x))dx}{\int_\Omega H(u(x))dx}, \quad c_2 = \frac{\int_\Omega f(x)(1 - H(u(x)))dx}{\int_\Omega (1 - H(u(x)))dx}$$

  and $\delta_\epsilon$ is regularized Dirac function. It is a derivative of

$$H_\epsilon(u) = \frac{1}{2}(1 + \frac{2}{\pi}tan^{-1}\left(\frac{u}{\epsilon}\right)).$$

- Notice, the first term is the curvature $\kappa$
- $u$ is changed only within narrow band where $\delta_\epsilon \neq 0$.

# Discrete Evolution Equation

The discrete evolution equation is:

$$u_{ij}^{k+1} = u_{ij}^{k} + \tau \delta_\epsilon(u_{ij}^{k}) \cdot \left[ \text{Curvature}(\mu) - \lambda_1 (f_{ij} - c_1)^2 + \lambda_2 (f_{ij} - c_2)^2 \right]$$

where $c_1$ and $c_2$ are average intensities of foreground and background at time $k\tau$ and Curvature($\mu$) is the curvature term applied at $u_{ij}$ (see previous Lecture).

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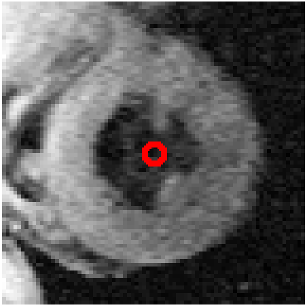
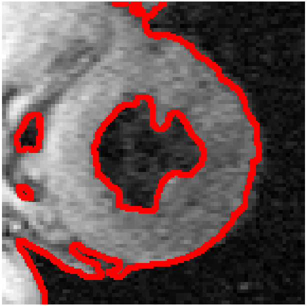
# Example: Comparison to GAC

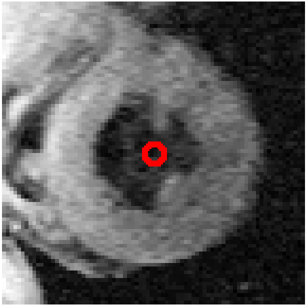


Initial contour

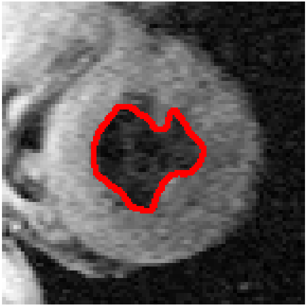


Result Chan-Vese
$\tau = 0.5, \lambda_1 = \lambda_2 = 0.05,$
$\epsilon = 10, \mu = 10, \nu = 0$

# Example: Comparison to GAC



Initial contour



Result GAC
$\tau = 0.25, c = 1, \epsilon = 0.5,$
$\beta = 0.5, p = 2, \sigma = 2.0$

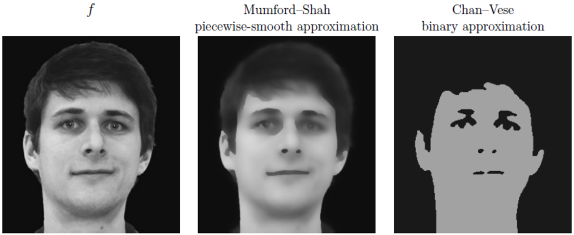
# Example: Comparison to Mumford-Shah

- Chan-Vese functional:

$$E_{CV}(\mathcal{C}, c_1, c_2) = \mu L(\mathcal{C}) + \nu A(\Omega_1) + \\ + \lambda_1 \int_{\Omega_1} |f(x) - c_1|^2 dx + \lambda_2 \int_{\Omega_2} |f(x) - c_2|^2 dx$$

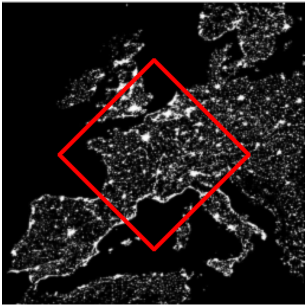
- Mumford-Shah functional:

$$E_{MS}(u, \mathcal{C}) := \lambda \int_{\Omega} (u - f)^2 dx + \beta \int_{\Omega \setminus K} |\nabla u|^2 dx + \mu |\mathcal{C}|$$

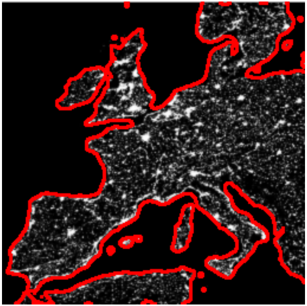



From [P. Getreuer, Chan–Vese Segmentation, IPOL Journal, 2012]

# Example



Initial contour

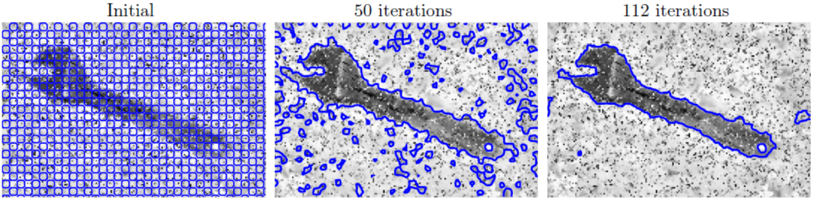


Result

$\tau = 0.5, \lambda_1 = 1, \lambda_2 = 0.02,$
$\epsilon = 10, \mu = 10, \nu = 0$

# Example: Initial $u_0$

- $u_0$ does not have to be SDF of some contour
- Good choice is $u_0(x, y) = \sin(\frac{\pi}{5}x)\sin(\frac{\pi}{5}y)$

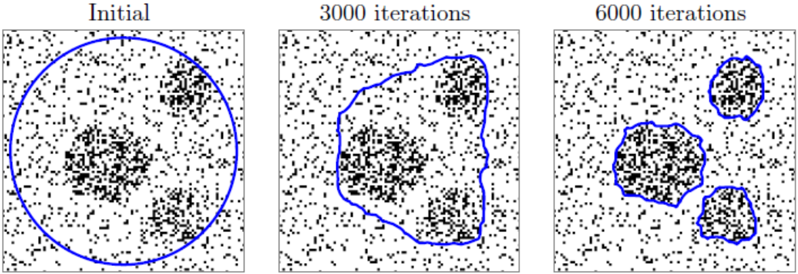



$$\tau = 0.5, \lambda_1 = 1, \lambda_2 = 1,$$
$$\mu = 0.2, \nu = 0, \epsilon = 1$$

From [P. Getreuer, Chan–Vese Segmentation, IPOL Journal, 2012]

# Example: Initial $u_0$

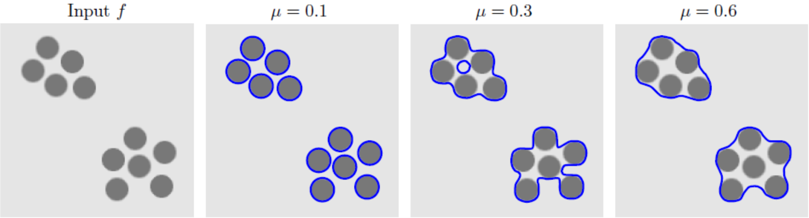
- $u_0$ is circle SDF




$$\tau = 0.5, \lambda_1 = 1, \lambda_2 = 1,$$
$$\mu = 0.3, \nu = 0, \epsilon = 1$$

From [P. Getreuer, Chan–Vese Segmentation, IPOL Journal, 2012]

# Example: Dependence on $\mu$

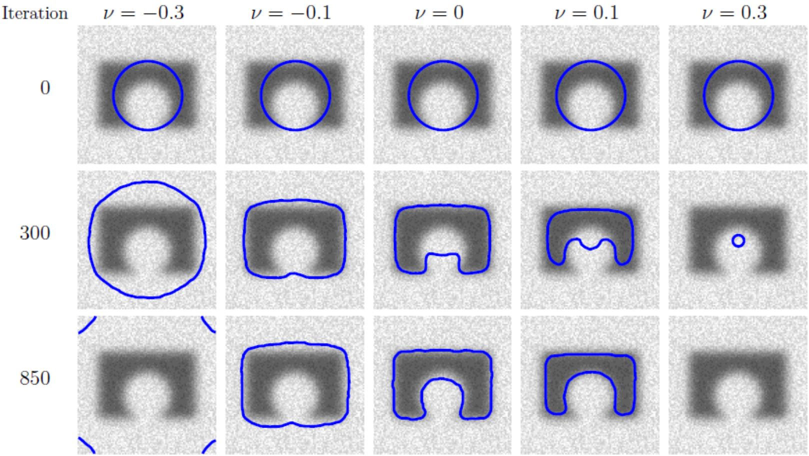



*Chan–Vese results with different values of $\mu$.*

$$\tau = 0.5, \lambda_1 = 1, \lambda_2 = 1,$$
$$\nu = 0, \epsilon = 1$$

From [P. Getreuer, Chan–Vese Segmentation, IPOL Journal, 2012]

# Example: Dependence on $\nu$

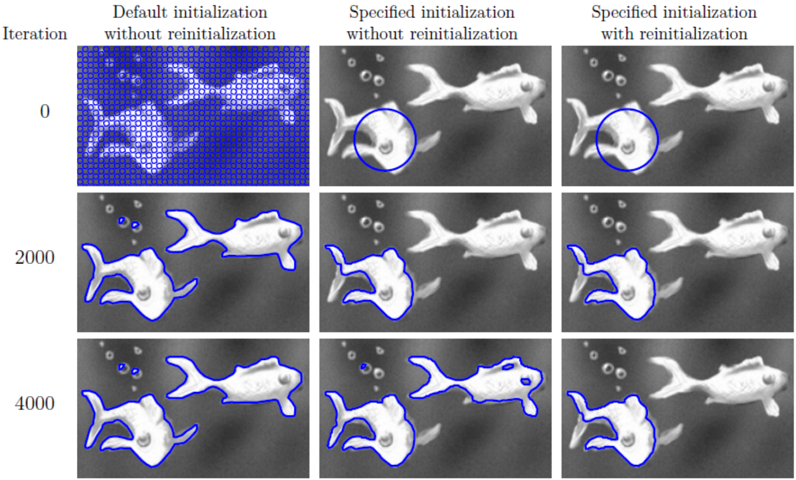



$$\tau = 0.5, \lambda_1 = 1, \lambda_2 = 1,$$
$$\mu = 0.2, \epsilon = 1$$

From [P. Getreuer, Chan–Vese Segmentation, IPOL Journal, 2012]

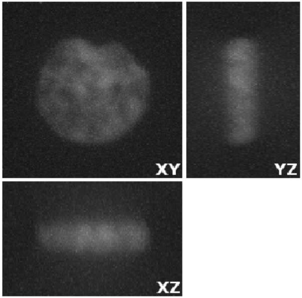
# Example: Reinitialization

- Reinitialization is required to avoid separated objects

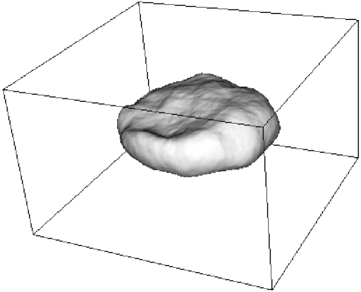



$$\tau = 0.5, \lambda_1 = 1, \lambda_2 = 1,$$
$$\mu = 0.15, \nu = 0, \epsilon = 1$$

# 3D Example




Input
Size: $160 \times 150 \times 60$



Result

# Contents

# Chan–Sandberg–Vese Model for Color Images

- Let $f(x) : \Omega \to \mathbb{R}^3$ is the input color image.
- Let $\mathcal{C}$ be the boundary between two regions $\Omega_1$ and $\Omega_2$.
  $\Omega = \Omega_1 \cup \Omega_2 \cup \mathcal{C}$
- Chan–Sandberg–Vese functional is defined

$$E_{CSV}(\mathcal{C}, c_1, c_2) = \mu L(\mathcal{C}) + \nu A(\Omega_1) +$$
$$+ \lambda_1 \int_{\Omega_1} ||f(x) - \mathbf{c}_1||^2 dx + \lambda_2 \int_{\Omega_2} ||f(x) - \mathbf{c}_2||^2 dx$$

  where all parameters are the same as in CV,
  $\mathbf{c}_1$ and $\mathbf{c}_2$ are mean colors of two distinct regions
- We minimize the functional with respect to $\mathbf{c}_1$, $\mathbf{c}_2$, and $\mathcal{C}$.

# Chan–Vese Model with Gabor Filters

- Gabor filters

$$G_{\sigma,F,\theta}(x,y) = \frac{1}{2\pi\sigma^2} e^{-\frac{x^2+y^2}{2\sigma^2}} \cdot e^{2\pi i F(x\cos\theta + y\sin\theta)},$$

  and

$$G_{\sigma,F,\theta}(x,y) = G_R + iG_I$$

- Image decomposed using Gabor filter bank for $N$ Gabors with different $(F,\theta,\sigma)$

$$\hat{f}_i = \sqrt{(G_{Ri} * f)^2 + (G_{Ii} * f)^2},\ i = 1,...,N$$

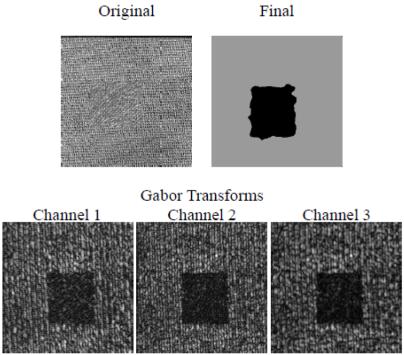
# Chan–Vese Model with Gabor Filters

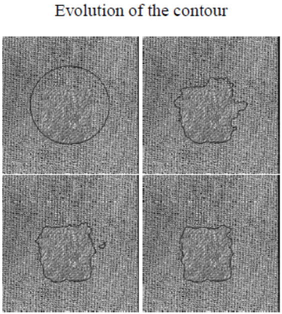
- [Sandberg B., Chan T., Vese L. A level-set and Gabor-based active contour algorithm for segmenting textured images. 2002.]
- Used to segment textured images
- Chan–Vese functional with Gabor filters is defined

$$E_{CVG}(\mathcal{C}, c_1, c_2) = \mu L(\mathcal{C}) + \nu A(\Omega_1) +$$
$$+ \int_{\Omega_1} \frac{1}{N} \sum_{i=1}^{N} \lambda_i |\hat{f}_i(x) - c_1^i|^2 dx + \int_{\Omega_2} \frac{1}{N} \sum_{i=1}^{N} \gamma_i |\hat{f}_i(x) - c_2^i|^2 dx$$

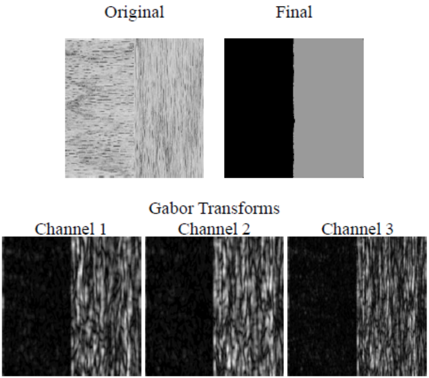
  all parameters are the same as in CV, $\mathbf{c}_1 = (c_1^1, ..., c_1^N)$ and $\mathbf{c}_2 = (c_2^1, ..., c_2^N)$ mean Gabor responses for two distinct regions
- We minimize the functional with respect to $\mathbf{c}_1$, $\mathbf{c}_2$, and $\mathcal{C}$.

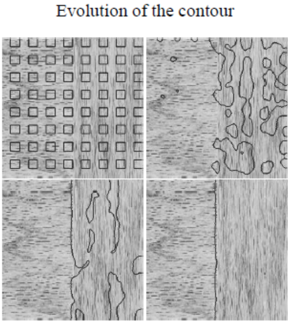
# Chan–Vese Model with Gabor Filters: Examples



# Chan–Vese Model with Gabor Filters: Examples



# Contents

# Summary

- Mumford-Shah functional is a general variational formulation of segmentation
  - It is mathematically very difficult (search for function $u$ and contours $\mathcal{C}$)
  - Non-convex
  - Computationally expensive
- Chan-Vese active contours without edges is a region based segmentation approach
  - Segmentation into 2 components
  - Considers homogenity of regions not only contours
  - It is a special case of Mumford-Shah functional
  - Fast computation based on level sets

# Contents

# Deep Learning and Traditional Model-based Segmentation

- DL generally require A LOT of annotated training data that is difficult to obtain (e.g. in medical images)
- DL is still opened for many mathematical explanations and theories, which may cause incomprehensible segmentation results
- Traditional models are used a lot in cooperation with DL - hybrid methods

# Deep Learning and Traditional Model-based Segmentation: Fresh References

Applications and hybrid methods:

- Assisting radiologists to annotate medical images (e.g.; tumor segmentation) [1]
- Post-processing method to refine the segmentation results that are generated by deep learning-based approaches (e.g. breast tumor [2], liver [3], and dental root [4] segmentation)
- Explicitly embedding shape information (e.g. a left ventricle shape model that is learned by autoencoder network can be embedded into the GAC [5])
- Reformulating the active contour model as a loss function to guide CNNs to learning richer features, such as Mumford–Shah loss [6], level set loss [7], and active contour loss [8]

# Fresh References for DL and Traditional Models


1 G. Unal, G. G. Slabaugh, T. Fang, S. Lankton, V. Canda, S. Thesen, and S. Qing, “System and method for lesion segmentation in whole body magnetic resonance images,” US Patent 8,155,405, Apr 2012.

2 Y. Hu, Y. Guo, Y. Wang, J. Yu, J. Li, S. Zhou, and C. Chang, “Automatic tumor segmentation in breast ultrasound images using a dilated fully convolutional network combined with an active contour model,” Medical physics, vol. 46, no. 1, pp. 215–228, 2019.

3 X. Guo, L. H. Schwartz, and B. Zhao, “Automatic liver segmentation by integrating fully convolutional networks into active contour models,” Medical physics, vol. 46, no. 10, pp. 4455–4469, 2019.

4 J. Ma and X. Yang, “Automatic dental root cbct image segmentation based on cnn and level set method,” in Medical Imaging 2019: Image Processing, vol. 10949, 2019, p. 109492N.

5 T. A. Ngo, Z. Lu, and G. Carneiro, “Combining deep learning and level set for the automated segmentation of the left ventricle of the heart from cardiac cine magnetic resonance,” Medical Image Analisys, vol. 35, pp. 159–171, 2017.

6 B. Kim and J. C. Ye, “Mumford–shah loss functional for image segmentation with deep learning,” IEEE TIP, vol. 29, pp. 1856–1866, 2019.

7 Y. Kim, S. Kim, T. Kim, and C. Kim, “Cnn-based semantic segmentation using level set loss,” in Winter Conference on Applications of Computer Vision (WACV), 2019, pp. 1752–1760.

8 X. Chen, B. M. Williams, S. R. Vallabhaneni, G. Czanner, R. Williams, and Y. Zheng, “Learning active contour models for medical image segmentation,” in CVPR, 2019, pp. 11 632–11 640.

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# References


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- Chan, Tony F., and Luminita A. Vese. "Active contours without edges." IEEE Transactions on image processing 10.2 (2001): 266-277.
- Vese, Luminita A., Le Guyader, Carole. Variational methods in image processing. CRC Press, 2015.