

Numerical Schemes Level Set Methods. Geodesic Active Contours.

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Active Contour Model

- ▶ The **deformable contour** (snake) is a mapping:

$$C(s) : [0, 1] \rightarrow \mathbb{R}^2, \quad s \mapsto C(s) = (x(s), y(s))^T.$$

- ▶ We define **energy functional** of the contour as

$$E_{snake}(C) = \int_0^1 E_{int}(C(s)) + E_{ext}(C(s)) ds,$$

where $E_{int}(C(s))$ is **internal energy** defined as

$$E_{int}(C(s)) = \alpha(s)|C'(s)|^2 + \beta(s)|C''(s)|^2$$

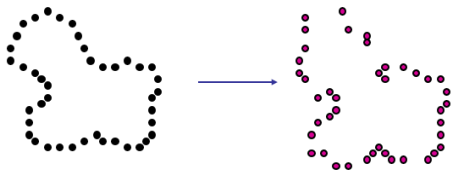
and $E_{ext}(C(s))$ is **external energy** defined as

$$E_{ext}(C(s)) = P(C(s)),$$

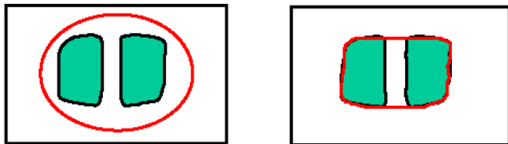
where P is the **potential** associated to the image.

Problems with Parametric Curves

- ▶ Reparametrization needed (hard with surfaces in 3D)



- ▶ Cannot handle topological changes



- ▶ Hard to extend to 3D

$$\begin{aligned} \gamma \frac{\partial \mathbf{v}}{\partial t} - \frac{\partial}{\partial s} \left(w_{10} \frac{\partial \mathbf{v}}{\partial s} \right) - \frac{\partial}{\partial r} \left(w_{01} \frac{\partial \mathbf{v}}{\partial r} \right) + 2 \frac{\partial^2}{\partial s \partial r} \left(w_{11} \frac{\partial^2 \mathbf{v}}{\partial s \partial r} \right) \\ + \frac{\partial^2}{\partial s^2} \left(w_{20} \frac{\partial^2 \mathbf{v}}{\partial s^2} \right) + \frac{\partial^2}{\partial r^2} \left(w_{02} \frac{\partial^2 \mathbf{v}}{\partial r^2} \right) + \nabla P(\mathbf{v}(s, r)) = 0, \end{aligned}$$

Level Sets Idea

- ▶ Curve \mathcal{C} is represented **implicitly** as a zero level set of a higher-dimensional function $u : \mathbb{R}^2 \rightarrow \mathbb{R}$.

$$\mathcal{C} = \{(x, y) : u(x, y) = 0\}$$

In **level set formulation**, the curve evolution according to

$$\frac{\partial \mathcal{C}}{\partial t} = \beta n$$

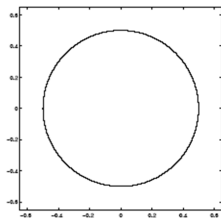
leads to the **evolution of embedding function** u according to

$$\frac{\partial u}{\partial t} + \beta |\nabla u| = 0$$

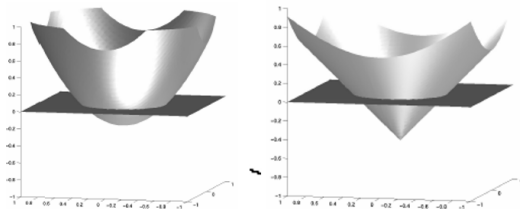
where n is curve normal and β is the evolution speed (scalar).

Level Sets Idea

- ▶ Curve



- ▶ Different embedding functions $u(x, y)$



Signed Distance Function (SDF): Example

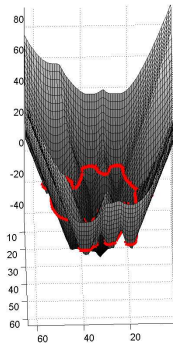
- ▶ For $C = \partial\Omega$, SDF d defined by:

$$d(\mathbf{x}) = \begin{cases} -\min_{\mathbf{y} \in C} |\mathbf{x} - \mathbf{y}| & \text{if } \mathbf{x} \in \Omega^- \\ +\min_{\mathbf{y} \in C} |\mathbf{x} - \mathbf{y}| & \text{if } \mathbf{x} \in \Omega^+ \cup C \end{cases}$$

- ▶ Normal $n = \nabla d$, $|\nabla d| = 1$
- ▶ Curvature $\kappa = \nabla \cdot \nabla d = \nabla^2 d = \Delta d$



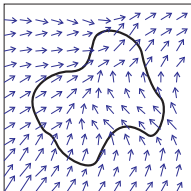
digital shape



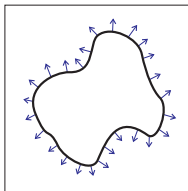
SDF $d(\mathbf{x})$

Three types of motion

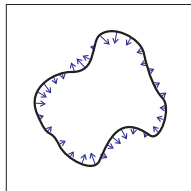
external velocity



normal motion



curvature motion



Level Set Evolution Equations

- ▶ **Contour motion** is driven by PDE

$$u_t + \beta |\nabla u| = 0$$

where β is the rate of evolution in the normal direction to the contour.

- ▶ There are three basic types of motion:
 - ▶ motion in the external **velocity field** $V(x, y, t)$

$$\beta = V(x, y, t) \cdot n$$

- ▶ motion in the **normal direction** (balloon force, dilation).

$$\beta = a$$

- ▶ motion involving mean **curvature** (internal force)

$$\beta = -\epsilon \kappa$$

- ▶ Different motions require different numerical schemes.

Level Set Evolution Equations: Normal Motion

- ▶ Normal motion:

$$u_t(t, \mathbf{x}) + a(\mathbf{x})|\nabla u(t, \mathbf{x})| = 0$$

$$u(0, \mathbf{x}) = u_0(\mathbf{x})$$

- ▶ If $a(\mathbf{x}) > 0$ we can transform to boundary value problem: Crossing times $T(\mathbf{x})$ of zero level set in all points by solving the Eikonal equation

$$a(\mathbf{x})|\nabla T(\mathbf{x})| = 1 ,$$

$$T(C_0) = 0, \text{ where } C_0 \text{ is initial contour}$$

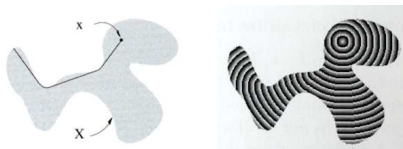
- ▶ Efficient numerical algorithm — Fast Marching Method ($\mathcal{O}(n \log n)$)

Normal Motion: Choice of $a(\mathbf{x})$

- ▶ Euclidean Distance (or SDF): $a(\mathbf{x}) = 1$

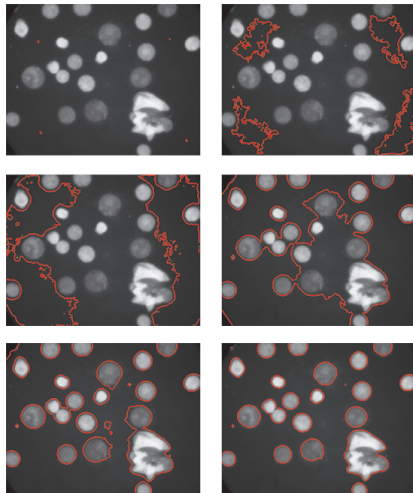


- ▶ Geodesic distance: $a(\mathbf{x}) = 1$ inside a mask and $a(\mathbf{x}) \rightarrow 0$ outside the mask



Normal Motion: Choice of $a(\mathbf{x})$

- ▶ Segmentation: $a(\mathbf{x}) = g(|\nabla f(\mathbf{x})|)$, where g is a decreasing function, e.g. $g = 1/(1 + \lambda|\nabla G_\sigma * f(\mathbf{x})|)$



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$$\beta = -\epsilon \kappa$$

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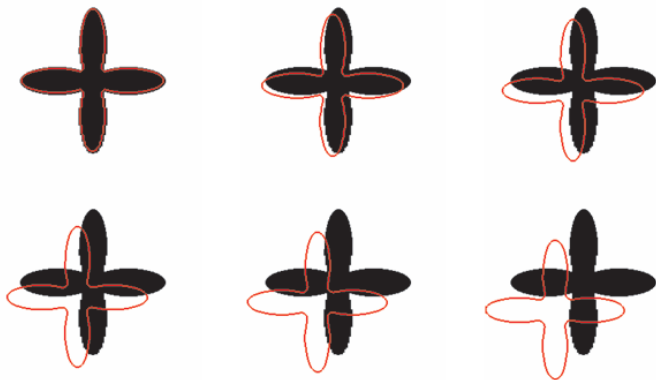
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Motion in the external velocity field



Motion in the external velocity field. The velocity vector is equal to $(-1, -1)$ for every grid point in this example.

Motion in the External Velocity Field

Suppose an (external) velocity field $V(x, y) = (v(x, y), w(x, y))$. Speed in normal direction is given by $\beta = V \cdot n$. Therefore, we have

$$u_t + (V \cdot n)|\nabla u| = u_t + V \cdot \frac{\nabla u}{|\nabla u|} |\nabla u| = 0$$

Then the evolution of interface embedded in implicit function u is described by convection (or advection) equation:

$$u_t + V \cdot \nabla u = 0$$

or we can write

$$u_t + v u_x + w u_y = 0$$

Numerical Solution

$$u_t + v u_x + w u_y = 0$$

- ▶ Time discretization – forward Euler

$$u_t = \frac{u_i^{n+1} - u_i^n}{\tau}$$

- ▶ Spatial discretization – upwind differencing dimension-by-dimension (shown for x axis)

$$\text{if } v > 0 \text{ then } u_x = \frac{u_i^n - u_{i-1}^n}{h} \quad \text{notation } D^{-x}$$

$$\text{if } v < 0 \text{ then } u_x = \frac{u_{i+1}^n - u_i^n}{h} \quad \text{notation } D^{+x}$$

$$\text{if } v = 0 \text{ then } 0$$

- ▶ More info in [Kalitkin N.N. Chislennyye metody.]

Numerical Solution

$$u_t + v u_x + w u_y = 0$$

has iterative **numerical scheme**

$$u_{ij}^{n+1} = u_{ij}^n - \tau \left(\begin{array}{l} \min(v_{ij}, 0) D_{ij}^{+x} + \max(v_{ij}, 0) D_{ij}^{-x} + \\ \min(w_{ij}, 0) D_{ij}^{+y} + \max(w_{ij}, 0) D_{ij}^{-y} \end{array} \right),$$

where

$$D_{ij}^{+x} = \frac{u_{i+1,j}^n - u_{ij}^n}{h}, \quad D_{ij}^{-x} = \frac{u_{ij}^n - u_{i-1,j}^n}{h}$$
$$D_{ij}^{+y} = \frac{u_{i,j+1}^n - u_{ij}^n}{h}, \quad D_{ij}^{-y} = \frac{u_{ij}^n - u_{i,j-1}^n}{h}$$

Higher order schemes exists.

Stability condition: $h/\tau > \max\{v, w\}$ for all grid points.

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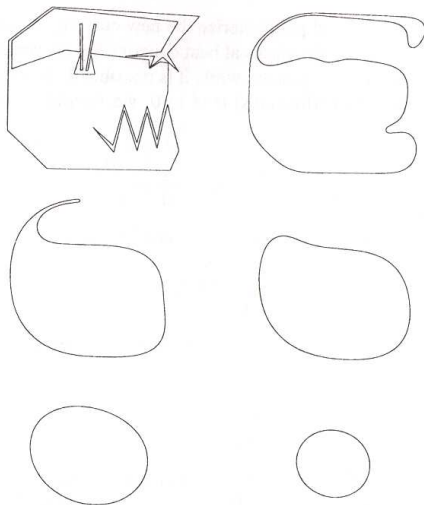
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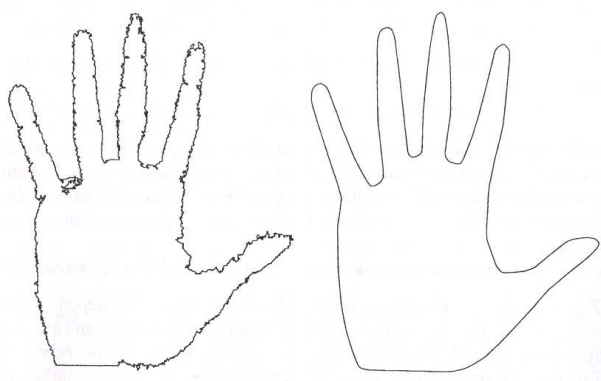
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Curvature Driven Motion, Example



Shape Smoothing



Mean Curvature Motion



Mean curvature motion ($\epsilon = 1.0$)

Curve Length and Shortening Flow

- ▶ Length of parametric curve can be computed as

$$L(C) = \int_0^1 |C'(q)| dq = \int_C ds$$

where $|C'(q)|$ is parametrization speed and ds is **arc-length**.

- ▶ It can be proved that the equation

$$\frac{\partial C}{\partial t} = \kappa \mathbf{n},$$

where κ is curvature and \mathbf{n} is curve normal, gives the fastest way to reduce L , i.e., moves the curve in the direction of the gradient of the functional L .

- ▶ This equation is known as the **Euclidean curve shortening flow**.

Mean Curvature Motion

Mean curvature motion is given by the speed $\beta = -\epsilon\kappa$, where $\epsilon > 0$ is constant and κ is the curvature.

Then the evolution equation is:

$$u_t = \epsilon\kappa|\nabla u|$$

Numerical scheme:

- ▶ Time discretization – forward Euler (first order)

$$u_t = \frac{u_i^{n+1} - u_i^n}{\tau}$$

- ▶ Spatial discretization – central differencing (second order) both for the gradient ∇u and the curvature

$$\kappa = \nabla \mathbf{n} = (u_x^2 u_{yy} - 2u_x u_y u_{xy} + u_y^2 u_{xx}) / |\nabla u|^3$$

$$u_x = \frac{u_{i+1}^n - u_{i-1}^n}{2h} \quad \text{notation } D^{0x}$$

Numerical Solution

Equation

$$u_t = \epsilon \kappa |\nabla u|$$

has iterative **numerical scheme**

$$u_{ij}^{n+1} = u_{ij}^n + \tau (\epsilon K_{ij}^n \sqrt{(D_{ij}^{0x})^2 + (D_{ij}^{0y})^2}),$$

where K_{ij}^n is central difference approximation to κ in point (x_i, y_j) and time $n\tau$.

Stability condition: $4\tau\epsilon < h^2$. Therefore, τ needs to be $\mathcal{O}(h^2)$, i.e. it is smaller (one order of magnitude) than in the previous case.

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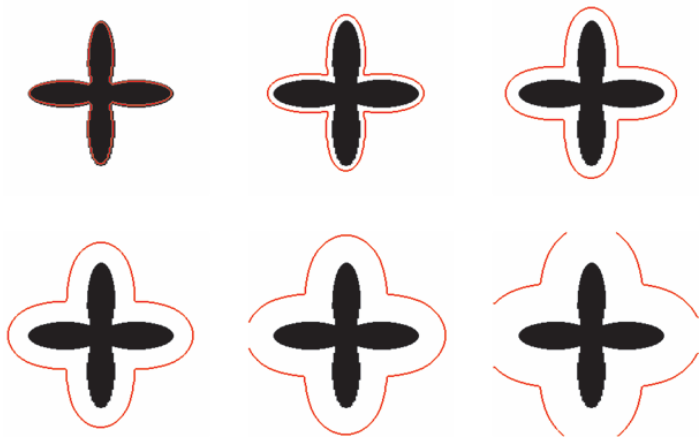
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Motion in the Normal Direction



Motion in the normal direction. $a = 1.0$.

Motion in the Normal Direction

Internally generated velocity field $\beta = a$

Then the evolution of u is described by the equation:

$$u_t + a|\nabla u| = 0$$

which leads to the **numerical scheme**

$$u_{ij}^{n+1} = u_{ij}^n - \tau[\max(a, 0)\nabla^+ + \min(a, 0)\nabla^-],$$

where

$$\nabla^+ = \left[\begin{array}{c} \min(D_{ij}^{+x}, 0)^2 + \max(D_{ij}^{-x}, 0)^2 \\ \min(D_{ij}^{+y}, 0)^2 + \max(D_{ij}^{-y}, 0)^2 \end{array} \right]^{1/2}$$

$$\nabla^- = \left[\begin{array}{c} \max(D_{ij}^{+x}, 0)^2 + \min(D_{ij}^{-x}, 0)^2 \\ \max(D_{ij}^{+y}, 0)^2 + \min(D_{ij}^{-y}, 0)^2 \end{array} \right]^{1/2}.$$

Stability condition: $\tau a < h$

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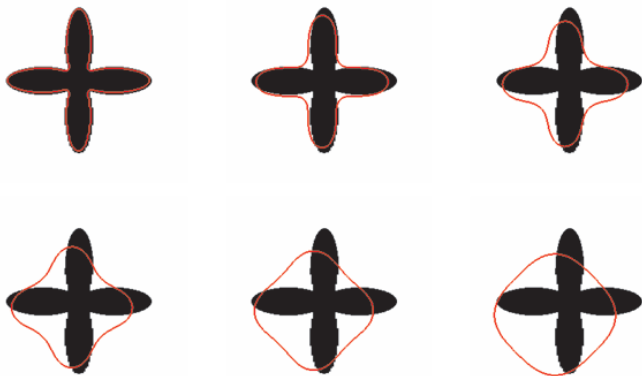
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All Types of Motion Together



All three types of motion together. All vectors in V are equal to $(-1, -1)$, $\epsilon = 0.25$ and $a = 1.0$.

All Types of Motion Together

The general equation

$$u_t + V \cdot \nabla u + a|\nabla u| = \epsilon\kappa|\nabla u|$$

is discretized with the **numerical scheme**

$$u_{ij}^{n+1} = u_{ij}^n - \tau \left[\begin{array}{c} [\max(a, 0)\nabla^+ + \min(a, 0)\nabla^-] + \\ \left[\begin{array}{c} \max(w_{ij}^n, 0)D_{ij}^{-x} + \min(w_{ij}^n, 0)D_{ij}^{+x} \\ + \max(v_{ij}^n, 0)D_{ij}^{-y} + \min(v_{ij}^n, 0)D_{ij}^{+y} \end{array} \right] - \\ -[\epsilon K_{ij}^n \sqrt{(D_{ij}^{0x})^2 + (D_{ij}^{0y})^2}] \end{array} \right],$$

Note: It's a combination of previous formulas.

Stability condition: The most restrictive (curvature) term forces $\tau = \mathcal{O}(h^2)$

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Active Contours: Snakes

- ▶ Active contour model (snakes) can be written as:

$$E_{AC}(C) = \int_0^1 \left(\alpha |C'(q)|^2 + \beta |C''(q)|^2 \right) dq + \int_0^1 P(C(q)) dq,$$

where P is the potential field.

- ▶ Often $P(C(q)) = g(|\nabla G_\sigma * I(C(q))|)$ where $g : [0, \infty) \rightarrow \mathbb{R}^+$ is a strictly decreasing function such that $g(r) \rightarrow 0$ as $r \rightarrow \infty$.
- ▶ Problem: (internal) energy **depends on curve parametrization**, i.e. it changes if we substitute $q = \phi(r)$, $\phi : [c, d] \rightarrow [0, 1]$, $\phi' > 0$
- ▶ This is an undesirable property, since parametrizations are **not related to the geometry** of the curve (or object boundary), but only to the velocity they are traveled.

Geodesic Active Contours

- ▶ Caselles et.al. 1995, Kichenassamy et al. 1995
- ▶ Curve with minimal **geodesic** length is searched.

$$\begin{aligned} E_{GAC}(C) &= \int_0^{L(C)} g(|\nabla G_\sigma * I(C(s))|) ds = \\ &= \int_0^1 g(|\nabla G_\sigma * I(C(q))|) |C'(q)| dq \end{aligned}$$

where $g : [0, \infty) \rightarrow \mathbb{R}^+$ is a strictly decreasing function such that $g(r) \rightarrow 0$ as $r \rightarrow \infty$, e.g.,

$$g(|\nabla G_\sigma * I(x, y)|) = \frac{1}{1 + |\nabla G_\sigma * I(x, y)|}$$

- ▶ The curve is attracted by image edges, where the weight $g(|\nabla G_\sigma * I(C(q))|)$ is small.
- ▶ Energy functional is not convex and therefore there are **several local minima**.

Geodesic Active Contours Advantages

- ▶ Do not depend on parametrization of curve \mathcal{C}
- ▶ Allow topology changes
- ▶ Curve shortening flow is embedded
- ▶ Fast computation with level sets
- ▶ Straightforward extension to 3D

Geodesic Active Contours, Evolution Equation

- ▶ **Evolution equation** (i.e. Euler-Lagrange equation with $\frac{\partial \mathcal{C}}{\partial t}$ on the left side) for geodesic active contours is

$$\frac{\partial \mathcal{C}}{\partial t} = (g\kappa - (\nabla g \cdot \mathbf{n})) \mathbf{n}$$

where \mathbf{n} is curve normal (vector) and κ is curvature (scalar) [see E. Sakhæe, “A Tutorial on Active Contours”, 2014]

- ▶ This equation can be rewritten in level-set framework.
- ▶ The curve is embedded in SDF u with evolution speed $\nu = g\kappa - \nabla g \cdot n$.

The evolution equation becomes

$$\frac{\partial u}{\partial t} = g\kappa|\nabla u| - \nabla g \cdot \nabla u$$

Geodesic Active Contours, Evolution Equation

- ▶ Curve with minimal geodesic length is searched.

$$E_{GAC}(C) = \int_0^1 g(|\nabla G_\sigma * I(C(q))|) |C'(q)| dq$$

$$g(|\nabla G_\sigma * I(x, y)|) = \frac{1}{1 + |\nabla G_\sigma * I(x, y)|}$$

- ▶ The evolution equation becomes

$$\frac{\partial u}{\partial t} = g\kappa |\nabla u| - \nabla g \cdot \nabla u$$

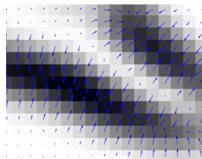
- ▶ Illustration



I



g



$-\nabla g$

Geodesic Active Contours, Evolution Equation

- ▶ The evolution equation

$$\frac{\partial u}{\partial t} = g\kappa|\nabla u| - \nabla g \cdot \nabla u$$

contains **curvature motion** and **velocity field motion**.

- ▶ It is solved using level set methods
- ▶ We can add **normal direction motion** (balloon force)

$$\frac{\partial u}{\partial t} = (c + \kappa)g|\nabla u| - \nabla g \cdot \nabla u$$

Geodesic Active Contours, Evolution Equation

- ▶ The most general form

$$\frac{\partial u}{\partial t} = (c + \epsilon\kappa)g|\nabla u| + \beta(\nabla P \cdot \nabla u)$$

where $P = |\nabla G_\sigma * I|$, c is a constant and

$$g(|\nabla G_\sigma * I(x, y)|) = \frac{1}{1 + |\nabla G_\sigma * I(x, y)|^p}$$

- ▶ Parameters:
 $\epsilon, \beta, c, \sigma$ and p

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Geodesic Active Contours, Iterative Scheme

- ▶ The equation

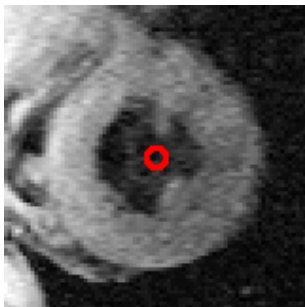
$$u_t = (c + \epsilon\kappa)g|\nabla u| + \beta\nabla P \cdot \nabla u$$

consists of three types of motion we have discussed before:

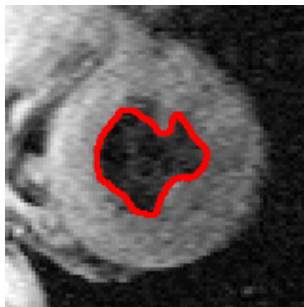
- ▶ **normal direction motion** with speed $cg(|\nabla G_\sigma * I(x, y)|)$
 - ▶ **curvature motion** multiplied by a factor $\epsilon g(|\nabla G_\sigma * I(x, y)|)$
 - ▶ **external velocity field motion** given by $\beta\nabla P$.
- ▶ Therefore, we have

$$u_{ij}^{k+1} = u_{ij}^k + \tau \cdot [\text{Normal}(cg) + \text{Curvature}(\epsilon g) + \text{Velocity}(\beta\nabla P)].$$

Geodesic Active Contours: Example



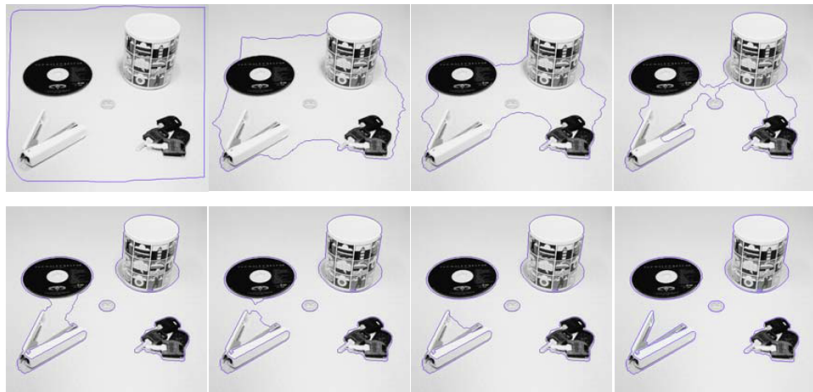
Initial contour



Result

$$\tau = 0.25, c = 1, \epsilon = 0.5,$$
$$\beta = 0.5, \rho = 2, \sigma = 2.0$$

Geodesic Active Contours: Example



Geodesic Active Contours: Example



Geodesic Active Contours: Example

▶ img15

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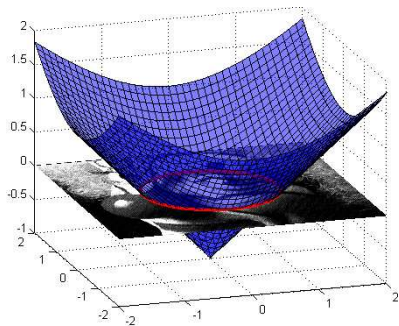
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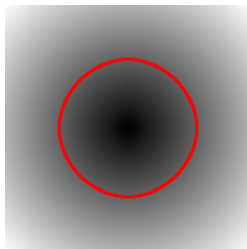
- ▶ Contour of interest \mathcal{C} (surface or curve) is the (zero) **level set** of an implicit function u , i.e. $\mathcal{C} = \{x : u(x) = 0\}$
- ▶ The **evolution** is steered by level set methods modifying u .



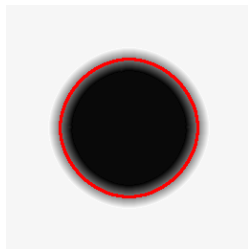
- ▶ We don't need to recompute u in the whole domain to track one contour.

General Procedure

- ▶ **Embed initial curve** into implicit function u as its zero level set. One can compute **signed distance function** using fast marching algorithm to get u for any contour.
- ▶ Solve the evolution equation in a **narrow band** around the contour of interest only.



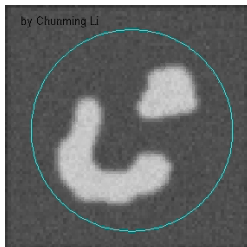
Whole domain



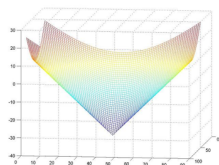
Narrow band

- ▶ We have to **reinitialize** the narrow band regularly.

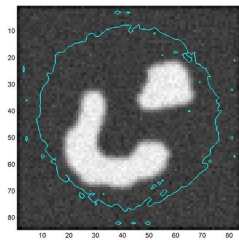
Reinitialization



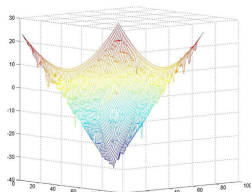
initial image



SDF



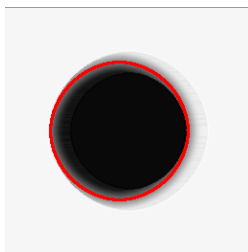
after 50 iterations



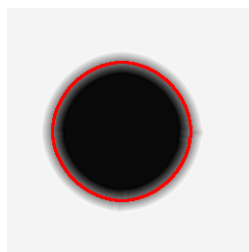
SDF

Narrow Band Reinitialization

- ▶ Before the contour leaves the narrow band we must reinitialize it.



Before reinitialization



After reinitialization

- ▶ Narrow band reinitialization procedure can be based on the **fast marching algorithm**.
- ▶ Maintaining sign distance function (SDF) around the contour **simplifies numerical formulas**.

Simplifications of Formulas

- ▶ Maintaining sign distance function around the contour **simplifies numerical formulas.**
- ▶ For SDF we have $|\nabla u| = 1$ and therefore the normal is

$$\mathbf{n}(\mathbf{x}) = \frac{\nabla u}{|\nabla u|} = \nabla u$$

and the curvature is

$$\kappa = \nabla \cdot \mathbf{n} = \nabla \cdot \nabla u = \nabla^2 u = \Delta u,$$

i.e. it is simply the Laplacian of u , i.e.

$$\kappa(\mathbf{x}) = \Delta u = u_{xx} + u_{yy} + u_{zz}.$$

Time complexity

- ▶ Narrow band method significantly reduces time complexity of the evolution procedure.
- ▶ Instead of $\mathcal{O}(n^{dim})$, where n is the number of grid points in one dimension and dim is the number of dimensions, we have time complexity $\mathcal{O}(kn^{dim-1})$ where k is narrow band width.
- ▶ The price is the necessity of regular band reinitialization.

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Summary

- ▶ We discussed numerical schemes for three different types of motion
- ▶ motion in the external velocity field
- ▶ mean curvature motion
- ▶ motion in the normal direction
- ▶ Curvature motion smooths the contour
- ▶ Geodesic active contours seeks curve with **minimal geodesic length**.
- ▶ We discussed active contours in the context of **level set methods**.
- ▶ Fast implementations of level set methods handle implicit function around zero level set only (e.g., **narrow band** method).

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References

- ▶ S. Osher, R. Fedkiw, Level Set Methods and Dynamic Implicit Surfaces, Springer 2003
- ▶ S. Osher, N. Paragios, Geometric Level Set Methods in Imaging, Vision, and Graphics, Springer 2003
- ▶ R. Kimmel, Numerical Geometry of Images: Theory, Algorithms, and Applications, Springer 2004
- ▶ E. Sakhaee, “A Tutorial on Active Contours” 2014