Лекция 6

Функции Габора и их применение

Dennis Gabor (1900 - 1979)

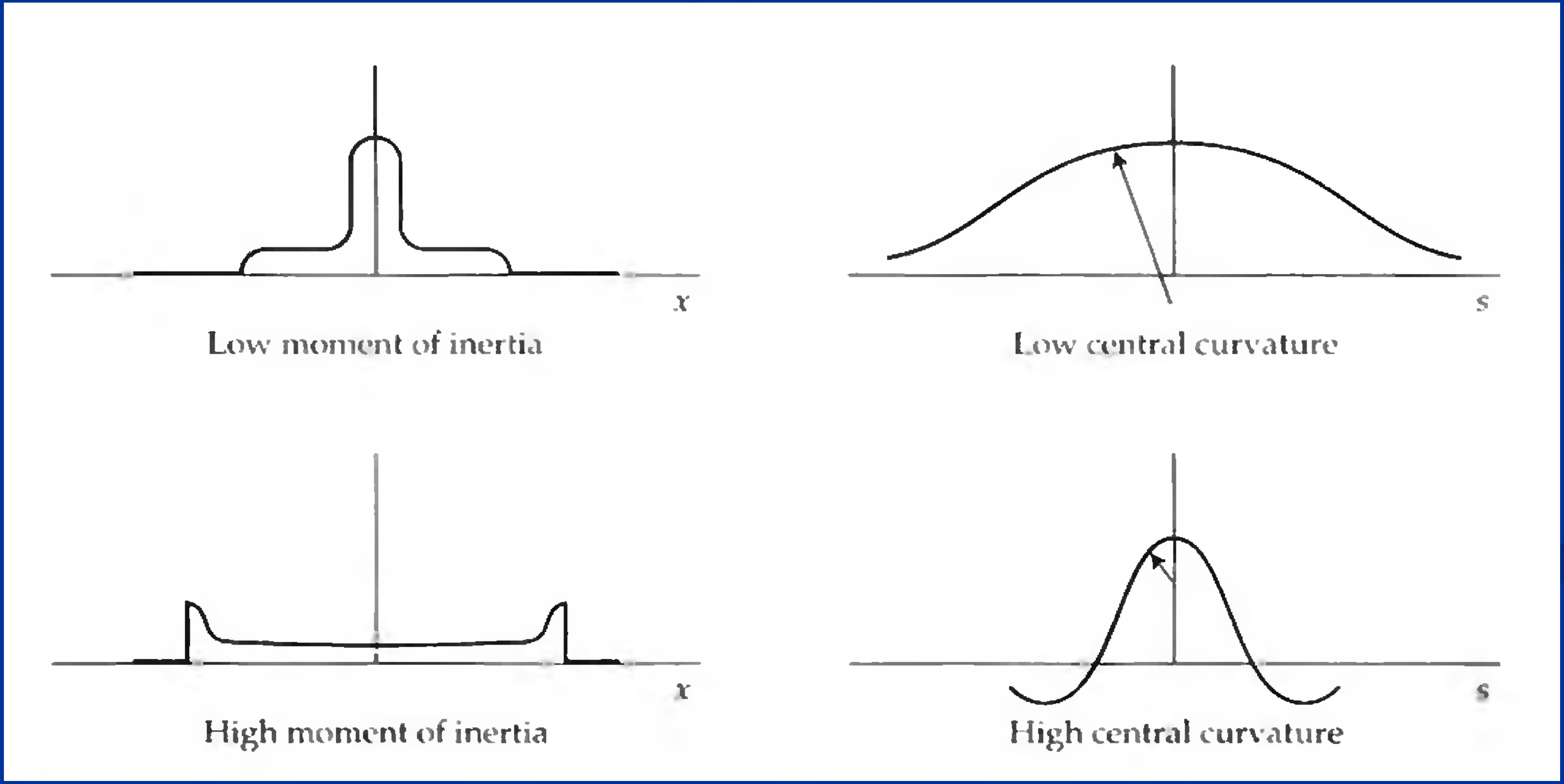
Hungarian-born electrical engineer who won the Nobel Prize for Physics in 1971 for his invention of holography, a system of lensless, three-dimensional photography that has many applications.

A research engineer for the firm of Siemens and Halske in Berlin from 1927, Gabor fled Nazi Germany in 1933 and worked with the Thomson-Houston Company in England, later becoming a British subject. In 1947 he conceived the idea of holography and, by employing conventional filtered-light sources, developed the basic technique. Because conventional light sources generally provided either too little light or light that was too diffuse, holography did not become commercially feasible until the demonstration, in 1960, of the laser, which amplifies the intensity of light waves.

In 1949 Gabor joined the faculty of the Imperial College of Science and Technology, London, where in 1958 he became professor of applied electron physics. His other work included research on high-speed oscilloscopes, communication theory, physical

Инерция (второй момент)

$$\int_{-\infty}^{\infty} x^2 f(x)dx$$



Дисперсия.

$$\sigma^2 = \langle (x - \langle x \rangle)^2 \rangle = \frac{\int\limits_{-\infty}^{+\infty} (x - \langle x \rangle)^2 f(x) \mathrm{d}x}{\int\limits_{-\infty}^{+\infty} f(x) \mathrm{d}x} = -\frac{F''(0)}{4\pi^2 F(0)} + \frac{(F'(0)^2)}{4\pi^2 (F(0))^2}.$$

$$\sigma_{f*g}^2 = \sigma_f^2 + \sigma_g^2.$$

Ширина локализации.

$$W_f = \frac{\int\limits_{-\infty}^{+\infty} f(x) \mathrm{d}x}{f(0)} = \frac{F(0)}{\int\limits^{+\infty} F(s) \mathrm{d}s} = \frac{1}{W_F}.$$

$$(\Delta x)^2(\Delta s)^2 = \frac{\int x^2 f f^* \, dx \int s^2 F F^* \, ds}{\int f f^* \, dx \int F F^* \, ds}$$

$$= \frac{\int xf.xf^* \, dx \int f' f'^* \, dx}{4\pi^2 \left(\int f f^* \, dx \right)^2}$$

$$\geq \frac{\left| \int (xf^*.f' + xf.f'^*) \, dx \right|^2}{16\pi^2 \left(\int f f^* \, dx \right)^2}$$

$$= \frac{\left| \int x \frac{d}{dx} (f f^*) \, dx \right|^2}{16\pi^2 \left(\int f f^* \, dx \right)^2}$$

$$= \frac{\left| \int f f^* \, dx \right|^2}{16\pi^2 \left(\int f f^* \, dx \right)^2}$$

$$= \frac{1}{16\pi^2}.$$

Некоторые неравенства. $|f(x)| \le \int\limits_{-\infty}^{+\infty} |F(s)|\mathrm{d}s$ выводится из $f(x) = \int\limits_{-\infty}^{+\infty} F(s)e^{i2\pi xs}\mathrm{d}s$. Аналогично $|f'(x)| \le 2\pi \int\limits_{-\infty}^{+\infty} |sF(s)|\mathrm{d}s$.

Неравенство Шварца. Для действительных функций

$$\left[\int g(x)f(x)\mathrm{d}x\right]^2 \le \int g^2(x)\mathrm{d}x \int f^2(x)\mathrm{d}x.$$

Для комплексных функций:

$$\left[\int (g^*(x)f(x) + g(x)f^*(x))\mathrm{d}x\right]^2 \le 4\int gg^*(x)\mathrm{d}x \int ff^*(x)\mathrm{d}x$$

Доказательство. Пусть ε -действительная константа

$$0 < \int (f(x) + \varepsilon g(x))(f(x) + \varepsilon g(x))^*\mathrm{d}x.$$

$$0 < \int f(x)f(x)^*\mathrm{d}x + \varepsilon \int (g^*(x)f(x) + g(x)f^*(x))\mathrm{d}x + \varepsilon^2 \int g(x)g(x)^*\mathrm{d}x.$$

В правой части неравенства стоит полином 2й степени относительно ε: $a + b\varepsilon + c\varepsilon^2$. для выполнения неравенства необходимо $b^2 - 4ac < 0$, откуда следует н-во Шварца.

$$f'(x) = -2bxf(x) \Rightarrow f(x) = ae^{-bx^2}$$

Фильтры Габора

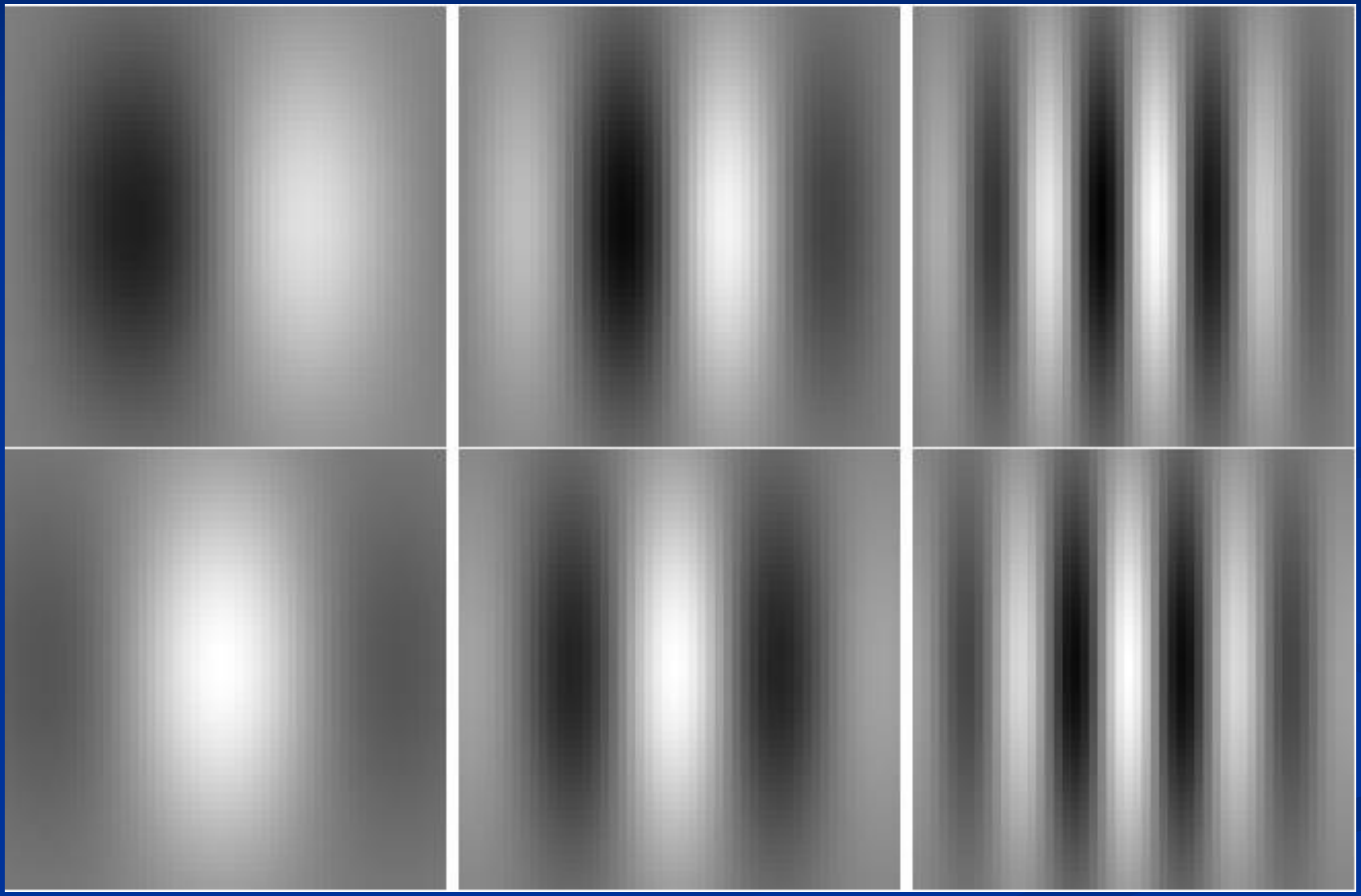
Gabor filters are formed by modulating a complex sinusoid by a Gaussian function:

$$g(x,y) = \overbrace{\frac{1}{2\pi\sigma_x\sigma_y}\exp\left(-\frac{1}{2}\left(\frac{\tilde{x}^2}{\sigma_x^2}+\frac{\tilde{y}^2}{\sigma_y^2}\right)\right)}^{\text{gaussian envelope}} \cdot \overbrace{\exp\left(2\pi j\omega\tilde{x}\right)}^{\text{complex sinusoidal}}$$

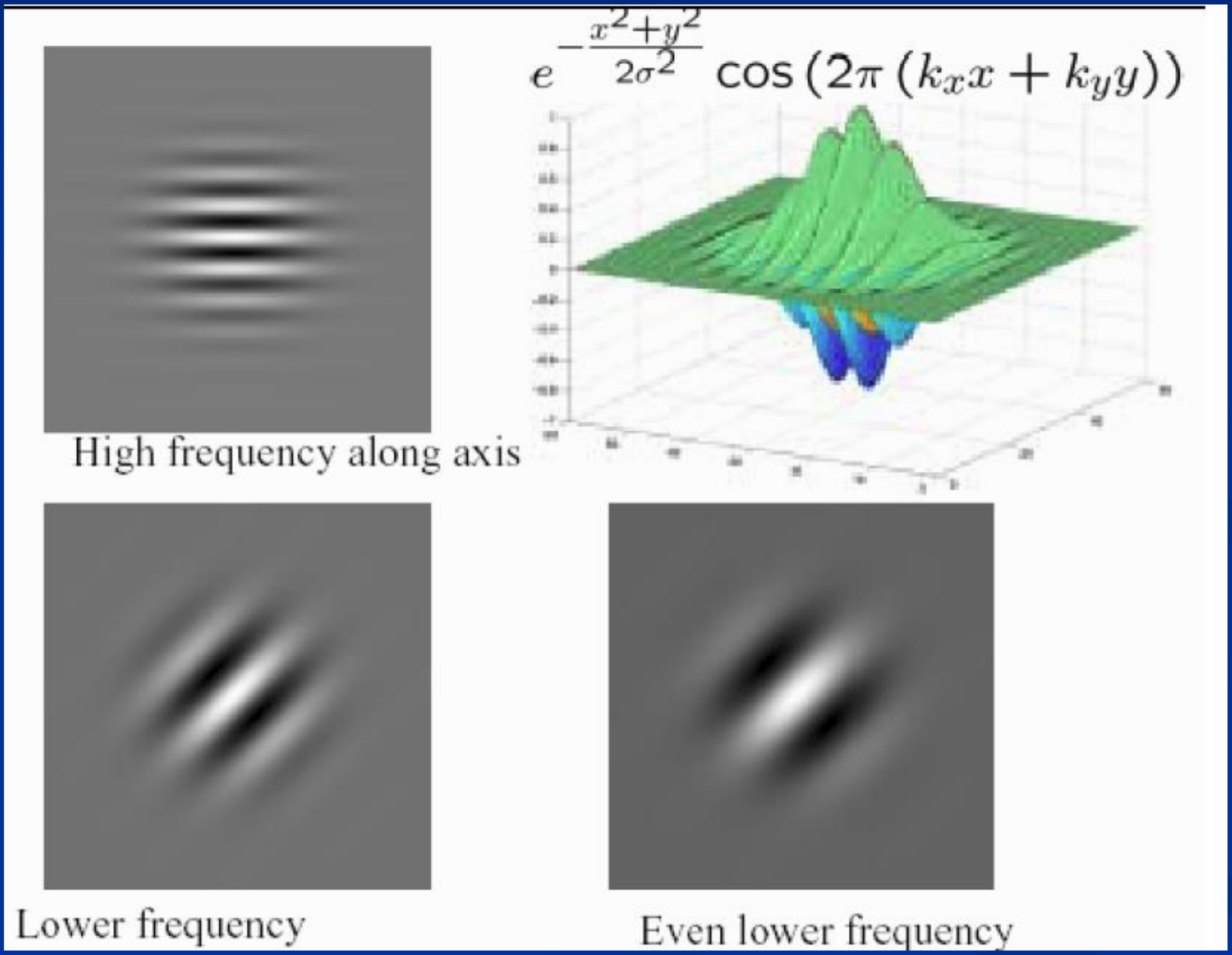
$$\text{with} \quad \begin{cases} \tilde{x} = x\cos(\theta) + y\sin(\theta) \\ \tilde{y} = -x\sin(\theta) + y\cos(\theta) \end{cases}$$

- σ_x and σ_y control spatial extent of filter
- θ is the orientation
- ω is the radial frequency of the sinusoid

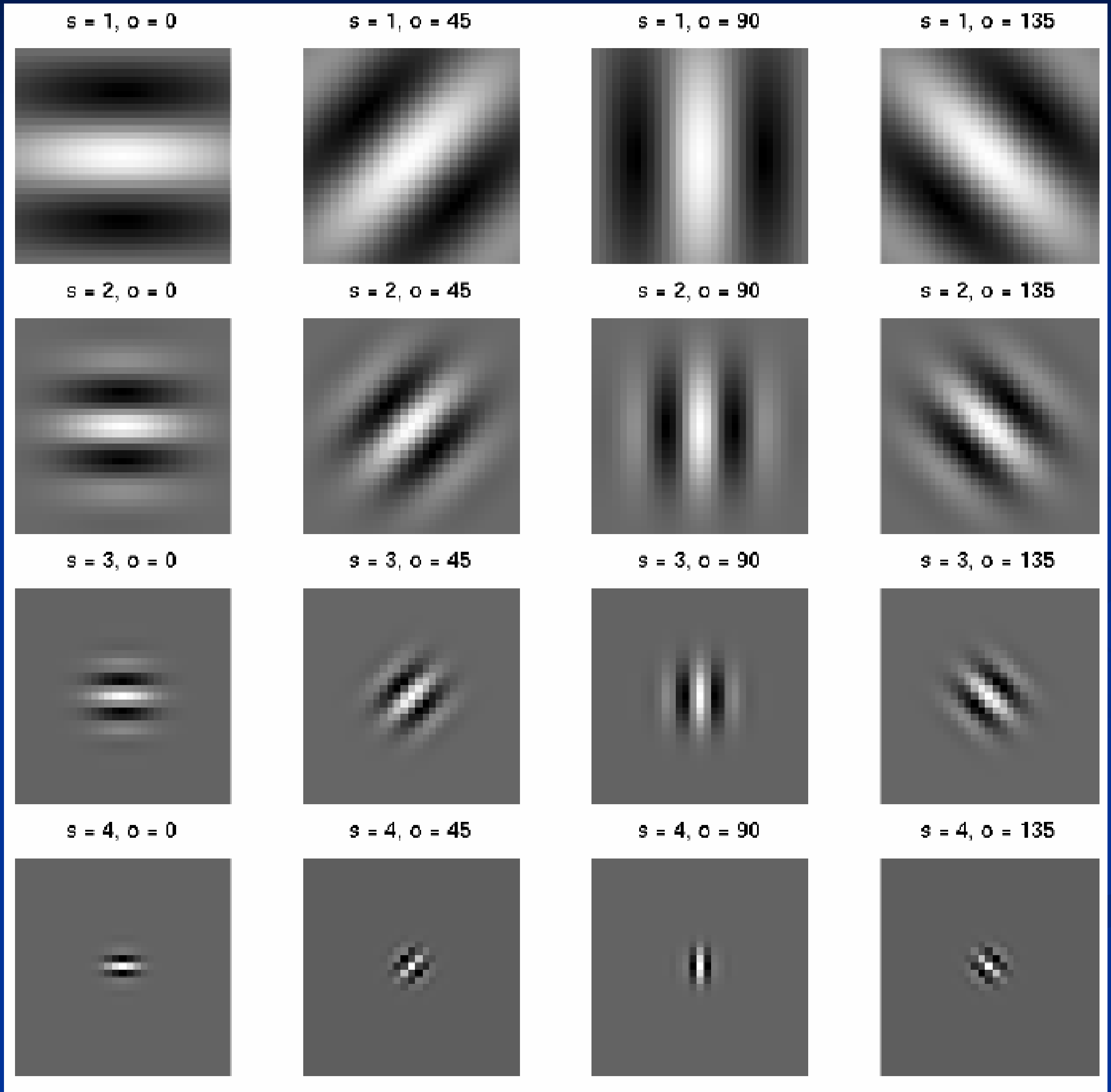
Фильтры Габора



Фильтры Габора



Фильтры Габора



Функции Габора

$$g(x, y) = \left(\frac{1}{2\pi\sigma_x\sigma_y}\right)\exp\left[-\frac{1}{2}\left(\frac{x^2}{\sigma_x^2} + \frac{y^2}{\sigma_y^2}\right) + 2\pi jWx\right],$$

$$G(u, v) = \exp\left\{-\frac{1}{2}\left[\frac{(u-W)^2}{\sigma_u^2} + \frac{v^2}{\sigma_v^2}\right]\right\},$$

where $\sigma_u = 1/2\pi\sigma_x$ and $\sigma_v = 1/2\pi\sigma_y$.

Габоровские вейвлеты

$$g_{mn}(x, y) = a^{-m} g(x', y'), \quad a > 1, \quad m, n = \text{integer}$$

$$x' = a^{-m}(x \cos\theta + y \sin\theta) \quad \text{and}$$

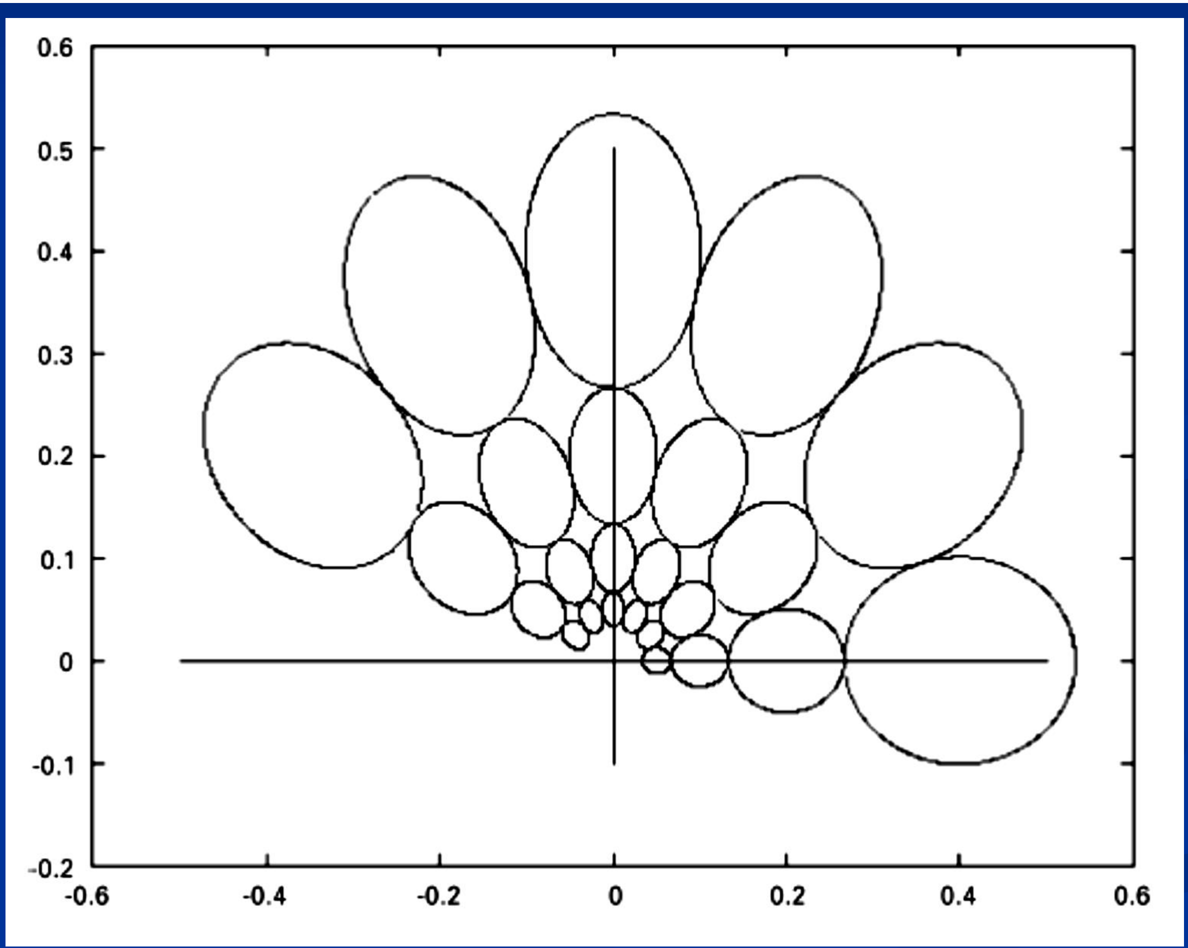
$$y' = a^{-m}(-x \sin\theta + y \cos\theta),$$

$$\theta = n\pi / K$$

K – number of orientations

Габоровские вейвлеты

$$W_{mn}(x, y) = \int I(x, y) g^*_{mn}(x - x_1, y - y_1)\, \mathrm{d}x_1\, \mathrm{d}y_1$$



$U_{\mathbf{h}} = 0.04$, $U_{\mathbf{l}} = 0.05$, $K = 6$ and $S = 4$

Габоровские вейвлеты

$$a = (U_{\mathbf{h}}/U_{\mathbf{l}})^{1/(S-1)}, \quad \sigma_u = \frac{(a-1)U_{\mathbf{h}}}{(a+1)\sqrt{2\ln 2}},$$

$$\sigma_v = \tan\left(\frac{\pi}{2k}\right)\left[U_{\mathbf{h}} - 2\ln 2\left(\frac{\sigma_u^2}{U_{\mathbf{h}}}\right)\right] \times \left[2\ln 2 - \frac{(2\ln 2)^2\sigma_u^2}{U_{\mathbf{h}}^2}\right]^{-1/2}$$

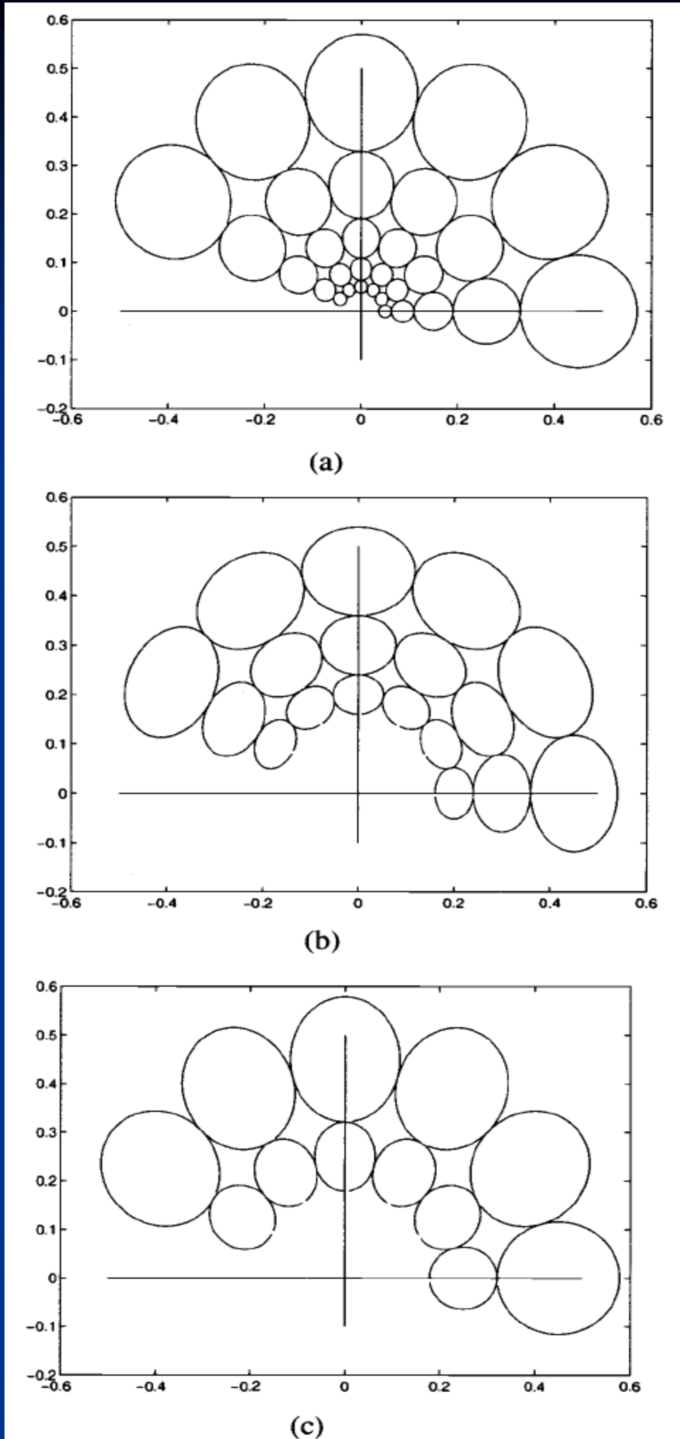
$$W = U_{\mathbf{h}} \text{ and } m = 0, 1, \ldots, S-1$$

K=6

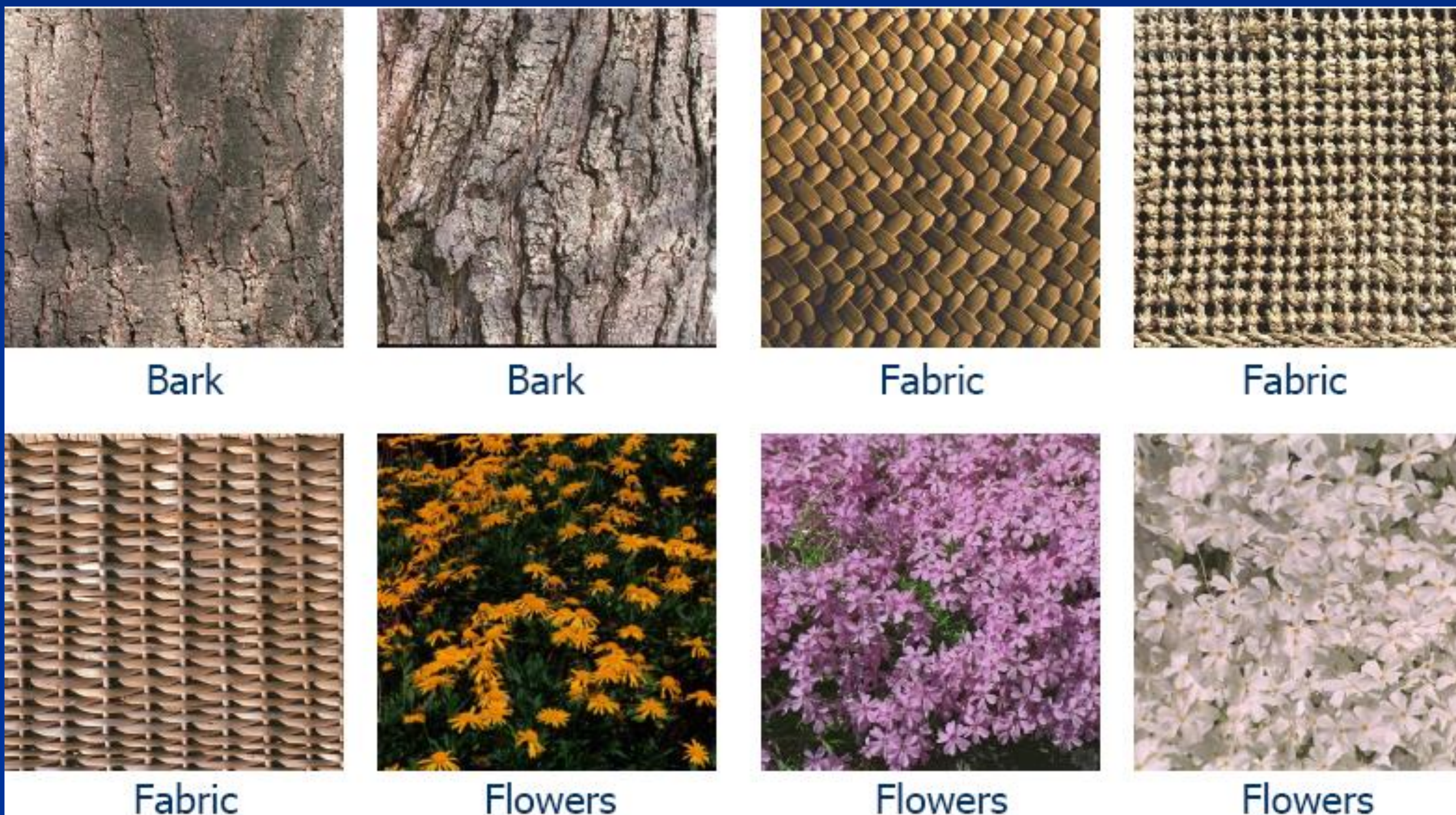
$a) \quad \sigma = 5, S = 5$

$b) \quad \sigma = 1.25, S = 3$

$c) \quad \sigma = 1, S = 2$

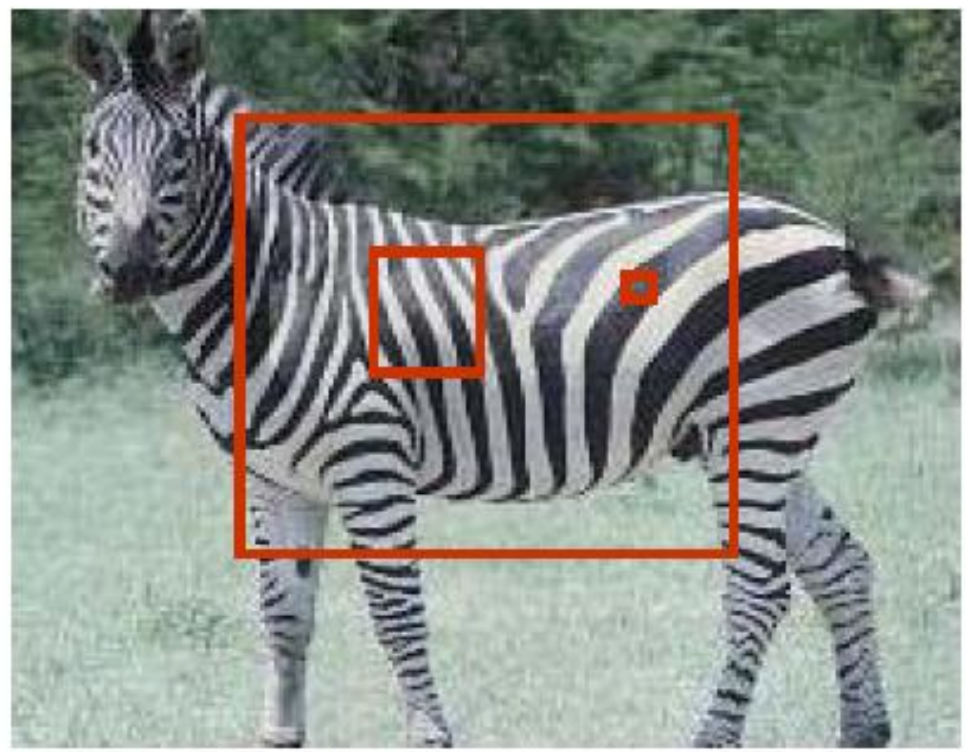


Текстуры



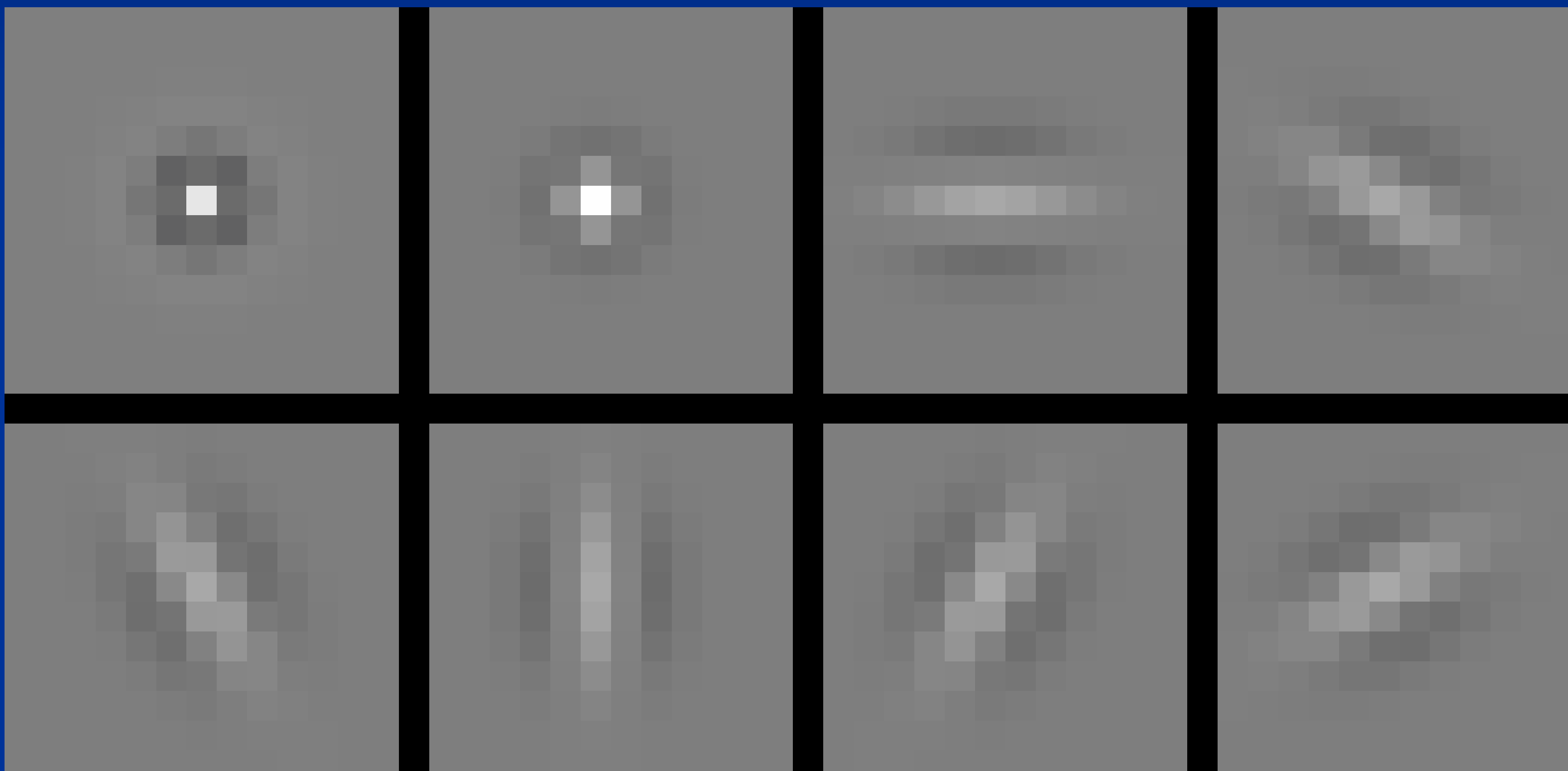
Текстуры

- Whether an effect is a texture or not depends on the scale at which it is viewed.

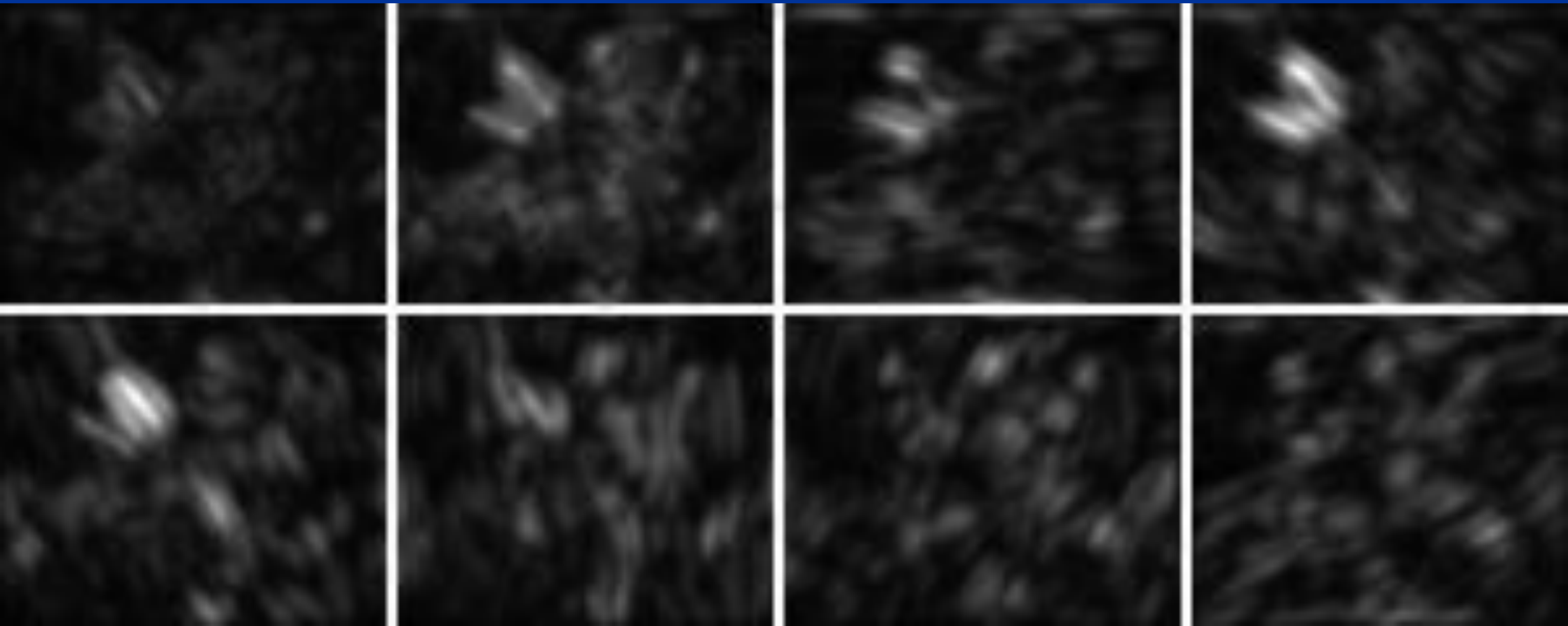


Текстуры

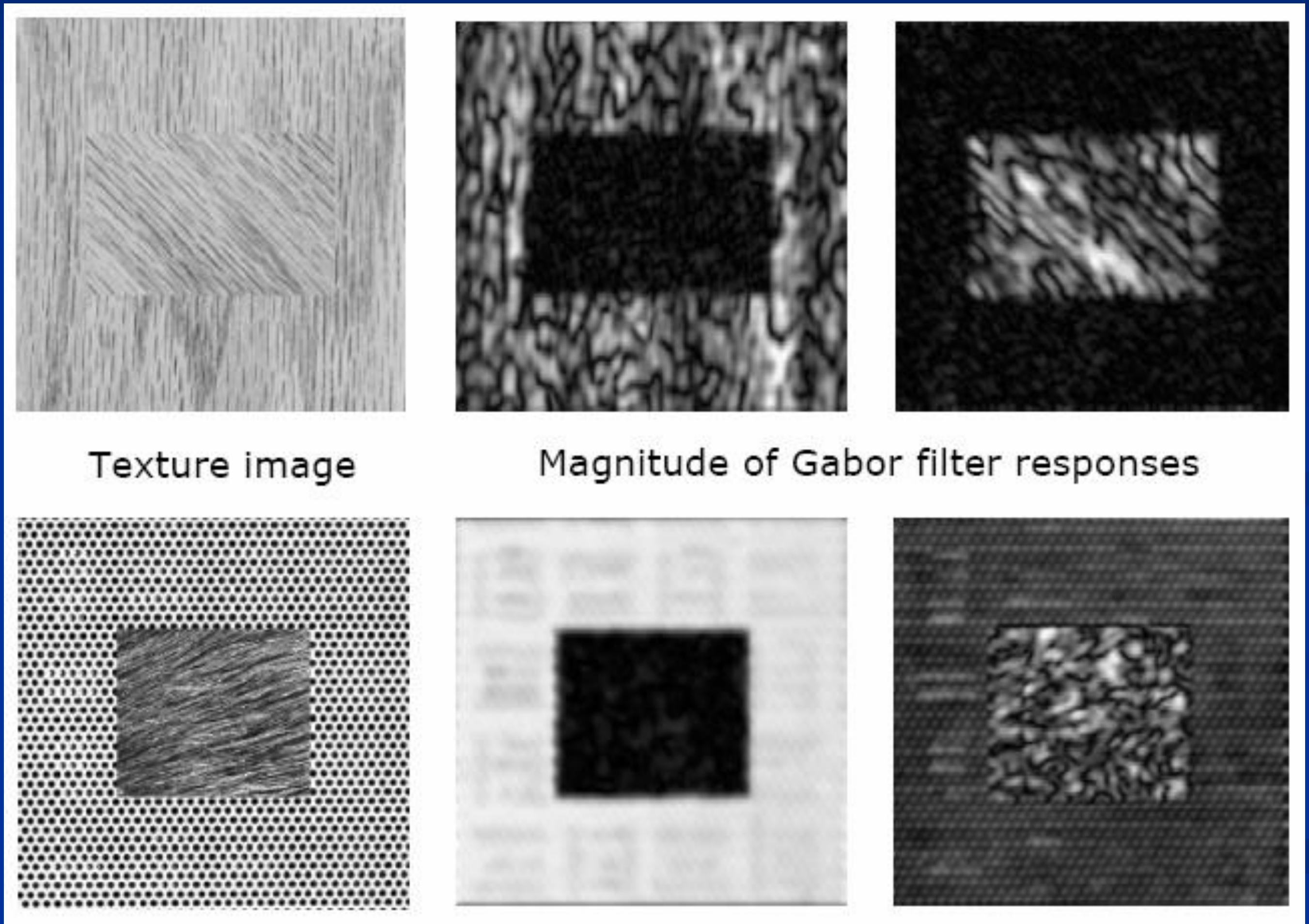
Банки фильтров



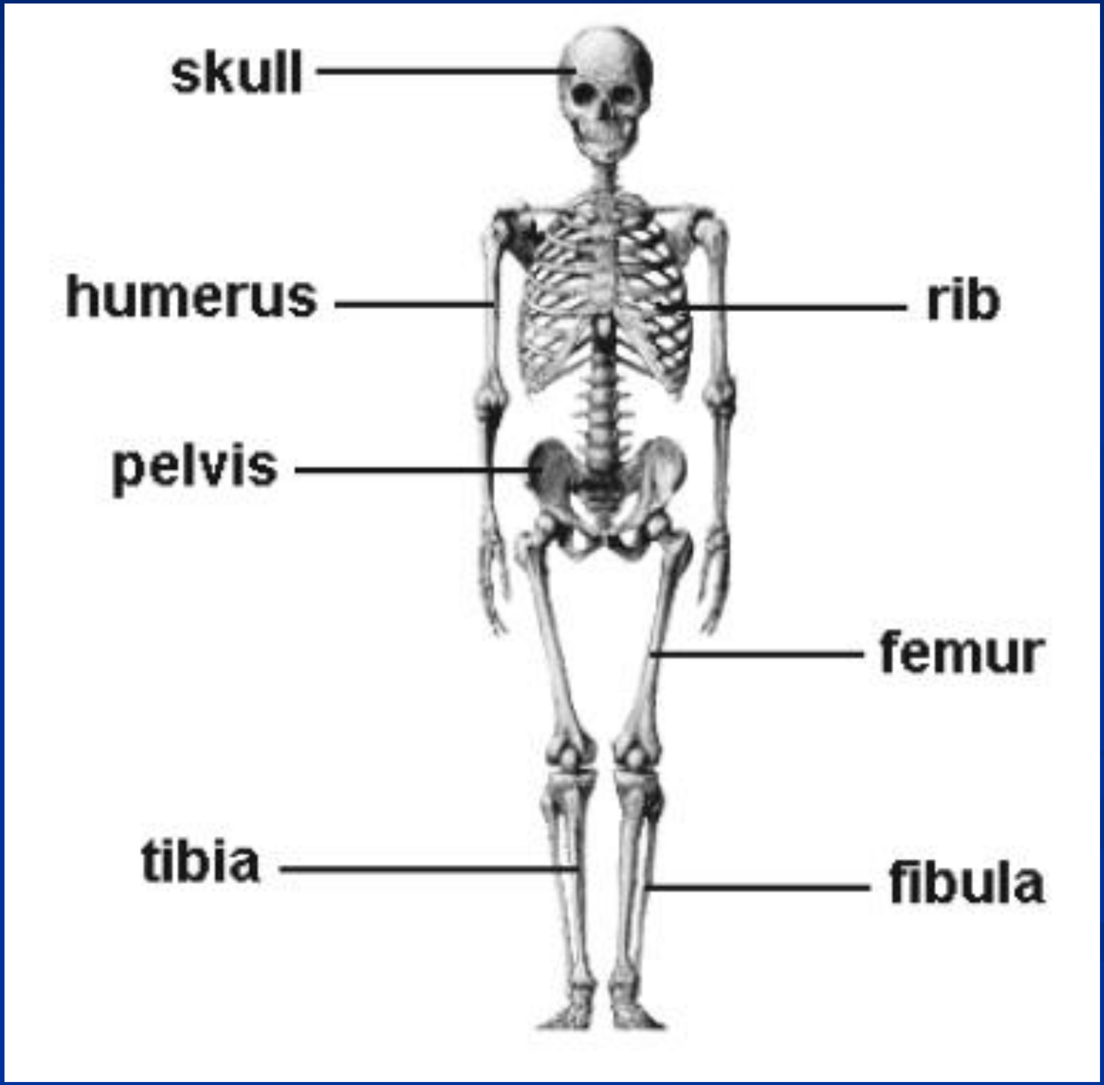
Банки фильтров



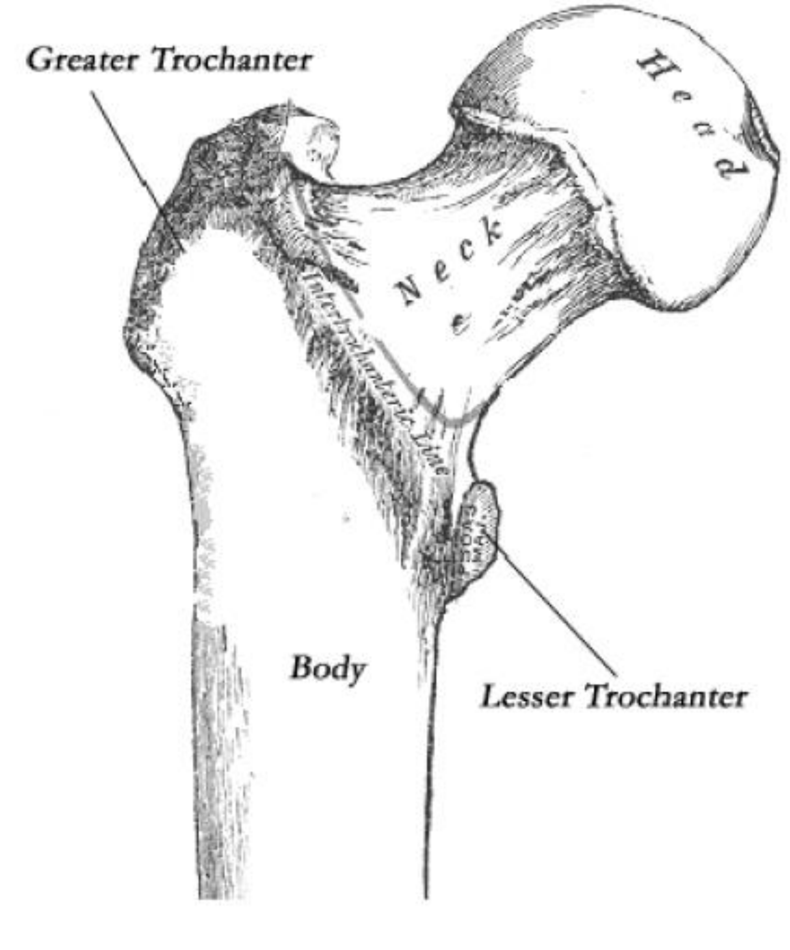
Фильтры Габора

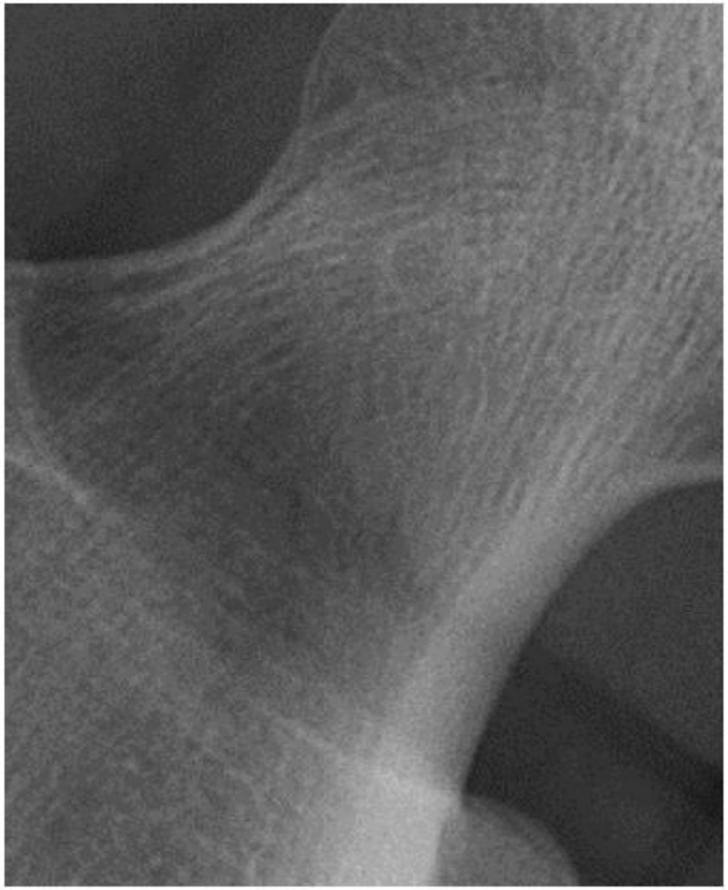


Анализ бедренной кости (femur)

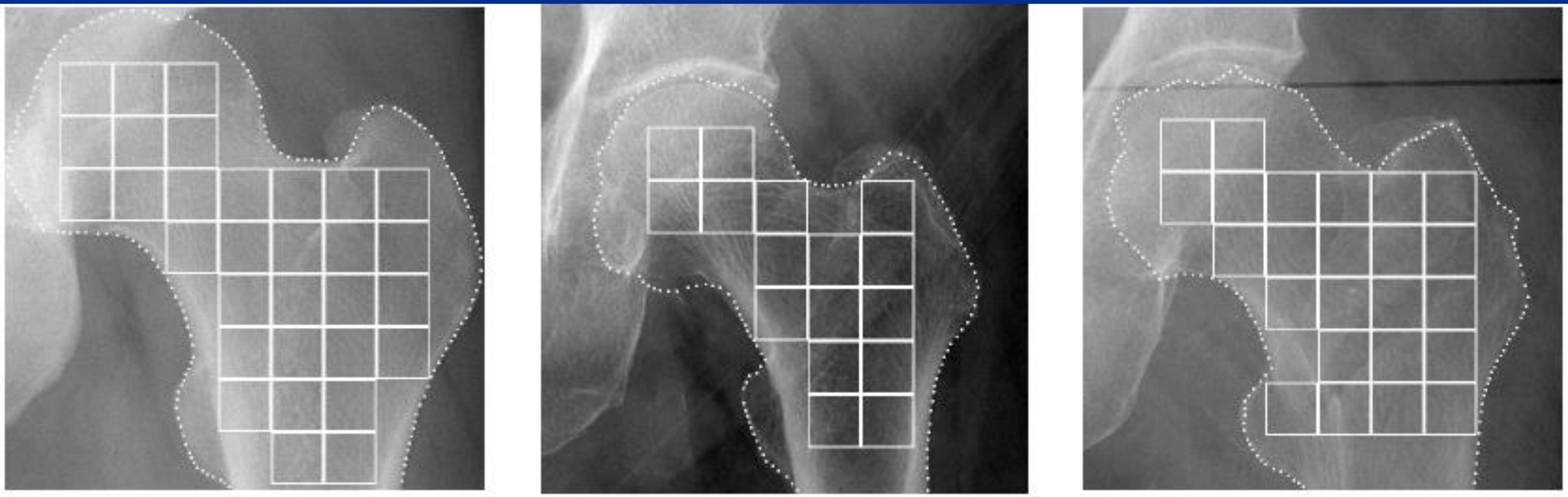


Шейка бедра


Greater Trochanter
Head
Neck
Intertrochanteric Line
Body
Lesser Trochanter



Шейка бедра



(a) (b) (c)

Figure 3.3. A femur is a natural structure that exhibits variations. (a) and (b) above are healthy, while (c) is fractured. Here, (a) is larger than (b). (c) has a relatively shorter neck, due to the fracture. Each grid square is a region sampled for texture orientation feature extraction

$$h(x,y) = g(x',y')\exp(2\pi jfx').$$

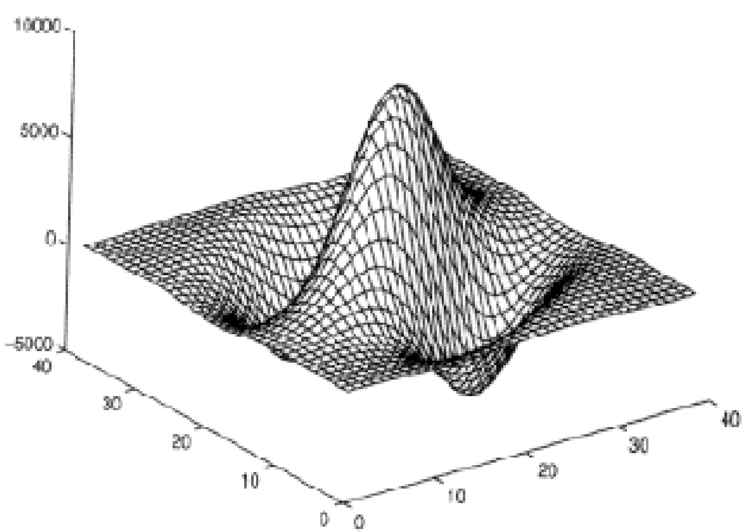
The oriented Gaussian function $g(x', y')$ is given by:

$$g(x',y') = \frac{1}{2\pi\lambda\sigma^2}\exp\left[-\frac{(x'/\lambda)^2 + y'^2}{2\sigma^2}\right],$$

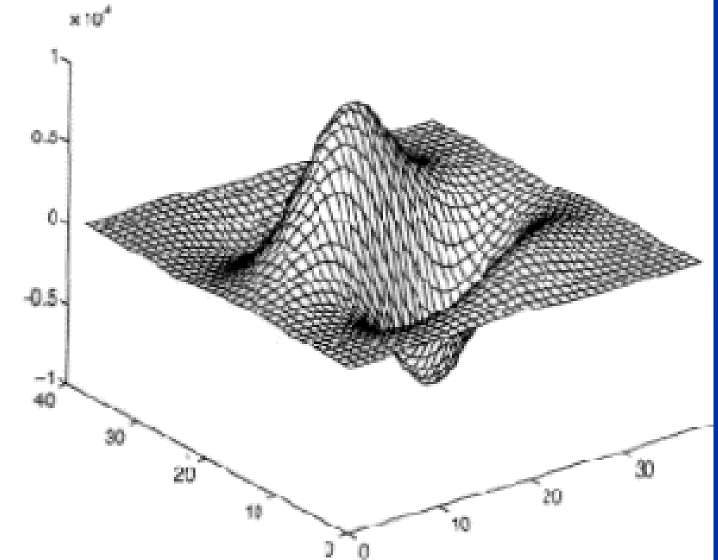
where $(x',y') = (x\cos\theta + y\sin\theta, -x\sin\theta + y\cos\theta)$ are rotated coordinates

$$h_{c,f\theta}(x,y) = g(x',y')\cos(2\pi fx'),$$

$$h_{s,f\theta}(x,y) = g(x',y')\sin(2\pi fx').$$



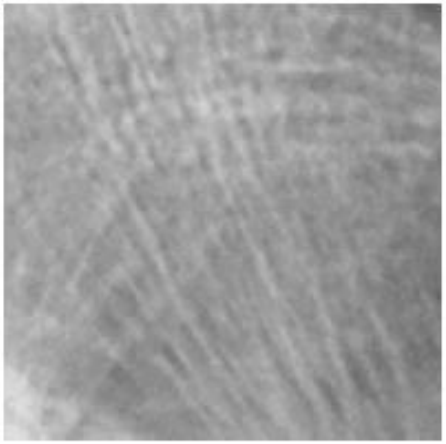
(a)



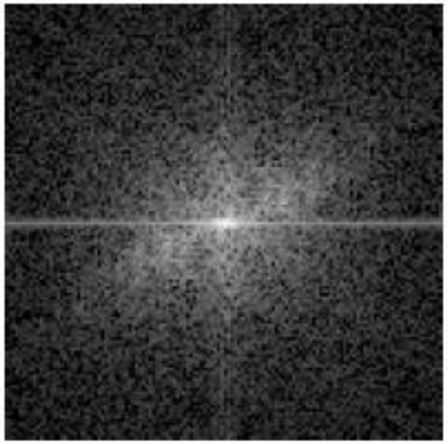
(b)

Фурье анализ текстуры кости

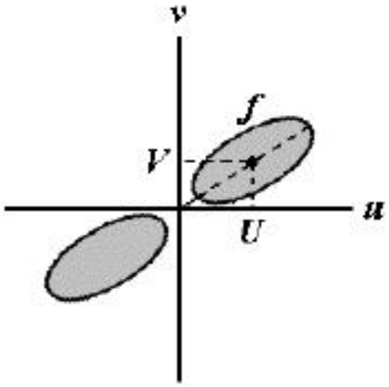
(a)



(b)



(c)



$$f_p = \sqrt{\overline{U}^2 + \overline{V}^2}$$

$f_p \cong$ 0.13 cycles per pixel

A Gabor filter bank of 1 frequency channel and 8 orientation channels is used to extract the orientation of the texture patterns in the femur. The centre frequency f of the Gabor filters is set to 0.13 cycles per pixel, and orientations range from 0° to 157.5°, incrementing in steps of 22.5°.

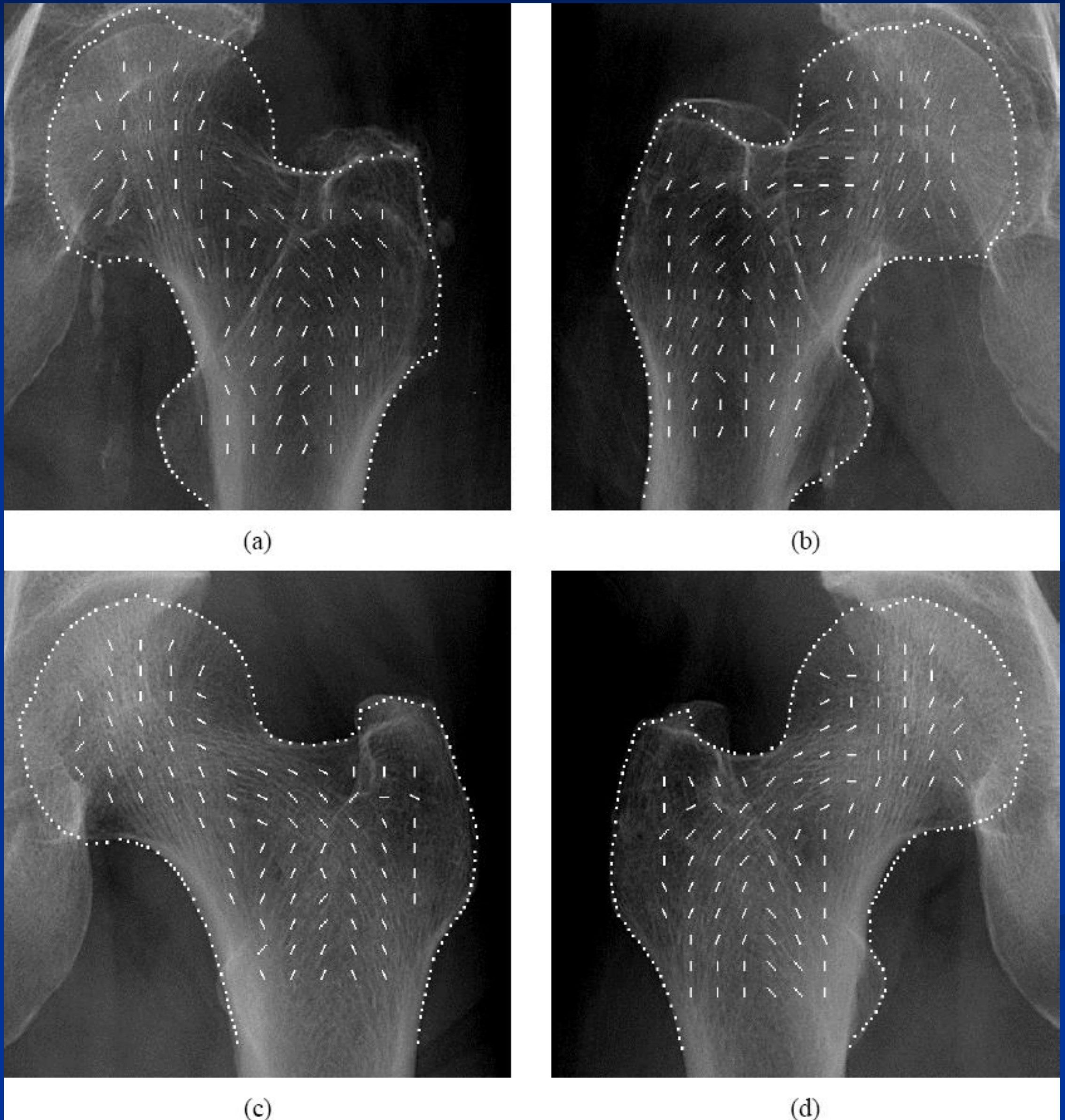
$$e_{c,f\theta}(x,y) = I(x,y) * h_{c,f\theta} \, ,$$

$$e_{s,f\theta}(x,y) = I(x,y) * h_{s,f\theta} \, .$$

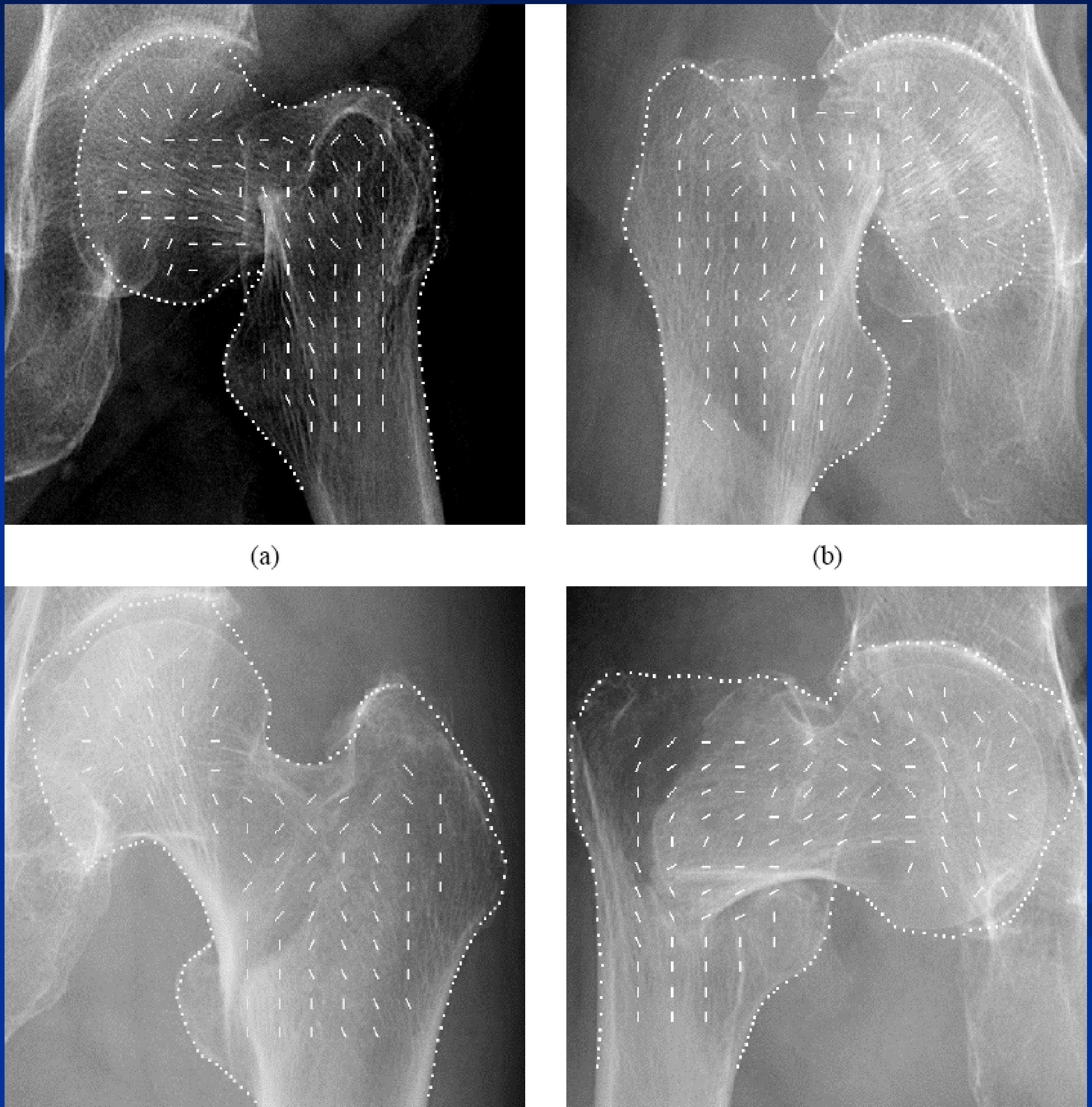
$$E_{f\theta}(x,y) = e^2_{c,f\theta}(x,y) + e^2_{s,f\theta}(x,y)$$

$$\overline{E}_{f\theta} = \frac{1}{S_x S_y} \sum_{x=1}^{S_x} \sum_{y=1}^{S_y} E_{f\theta}(x,y)$$

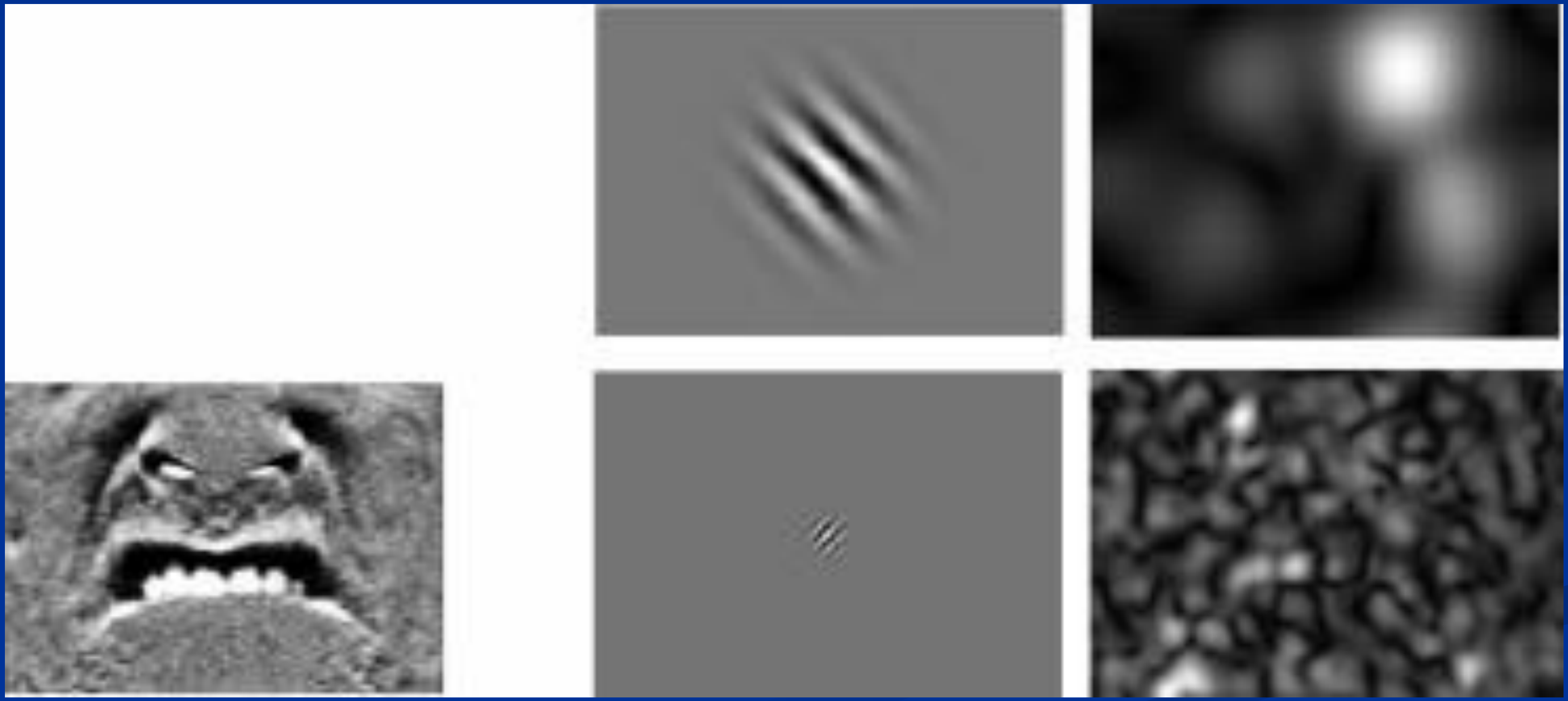
Целые кости



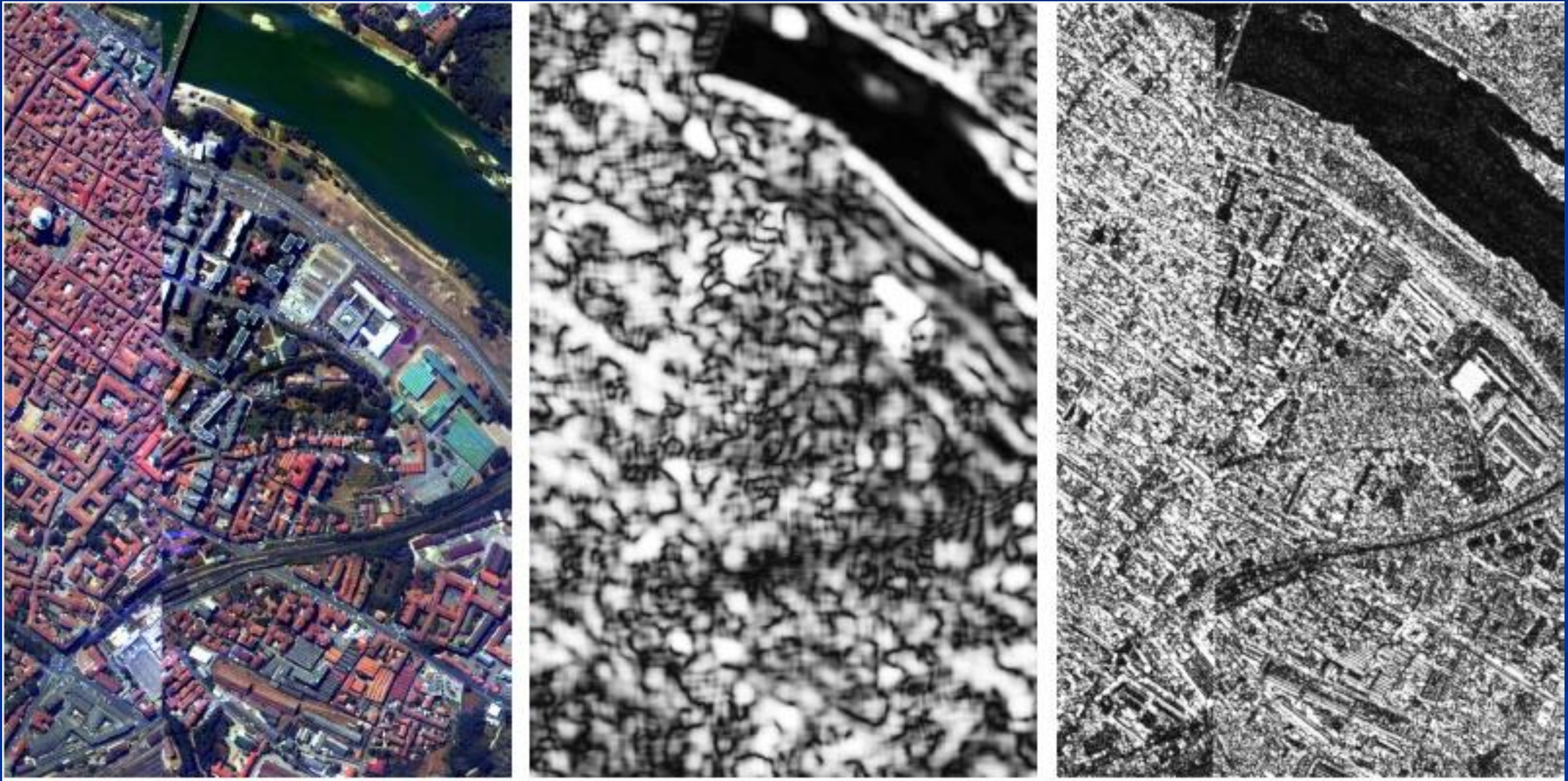
Кости с переломом



Фильтры Габора

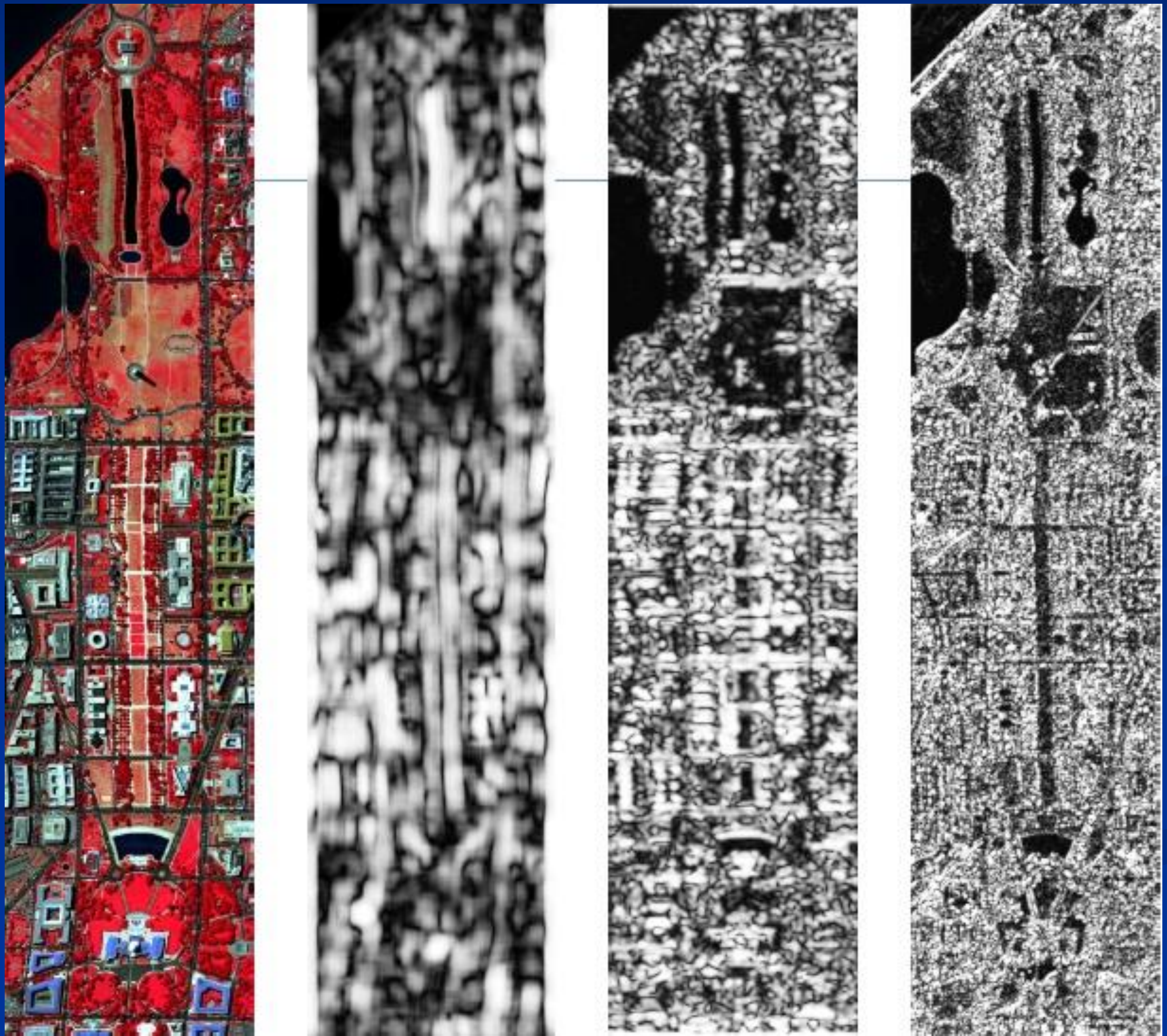


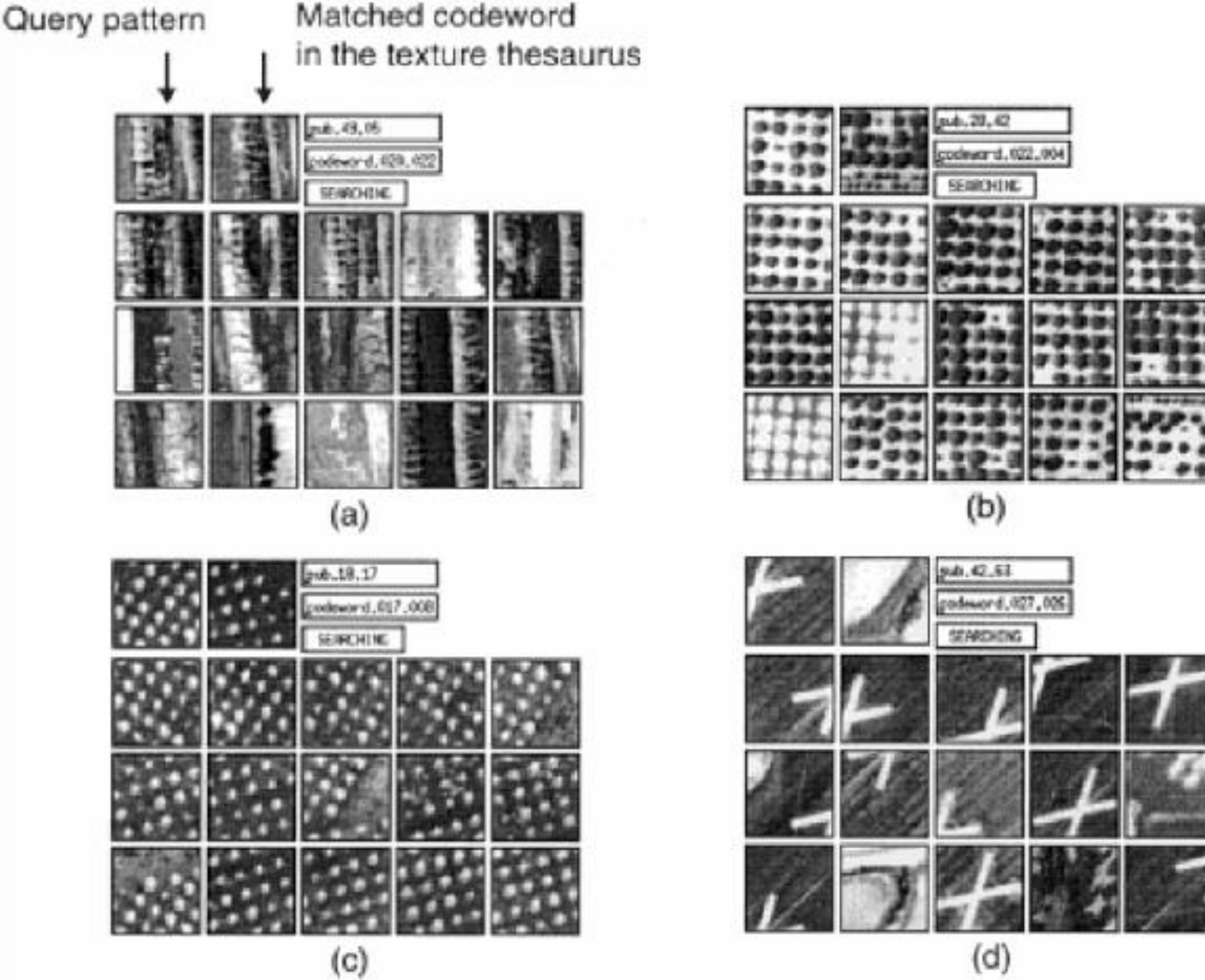
Фильтры Габора

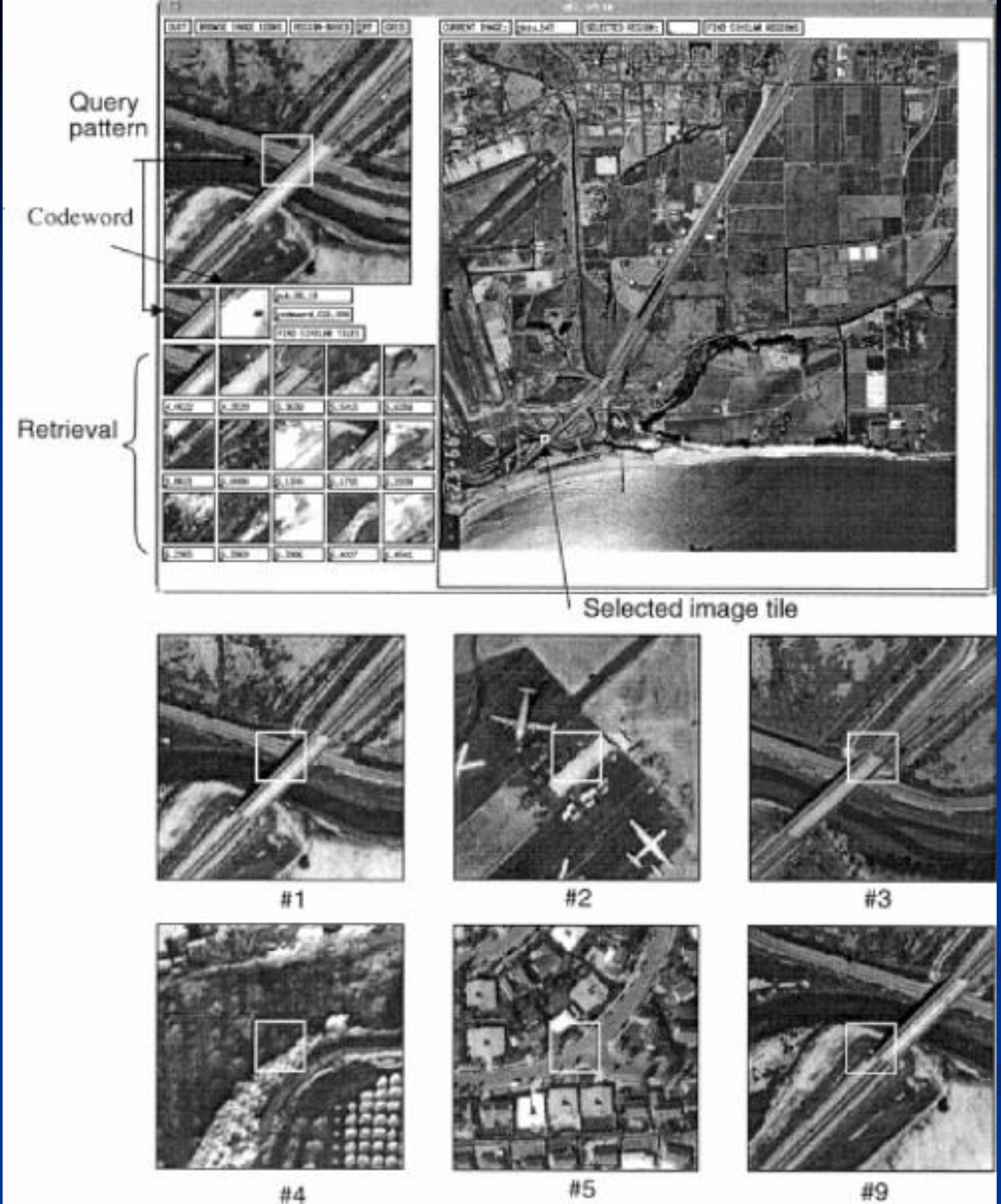


Gabor filter responses for a satellite image.

Фильтры Габора




Query pattern
Matched codeword
in the texture thesaurus
(a)
(b)
(c)
(d)



Full resolution of some retrieved tiles