Empirical Mode Decomposition

$f(t)$ – исходный сигнал

EMD представляет собой разложение следующего вида:

$$f(t) = r(t) + \sum_{i=1}^{N} \psi_i(t)$$

$r(t)$ – некая монотонная функция, называемая остатком

$\psi_i(t)$ – множество функций различной частоты, называемых Intrinsic Mode Function (IMF)

- Особенность: для разложения не используется фиксированный базис.
  $\psi_i(t)$ и $r(t)$ определяются самим сигналом.

Определение IMF

Будем называть функцию Intrinsic Mode Function, если:

1. Количество локальных экстремумов не более чем на единицу отличается от количества нулей функции на рассматриваемом промежутке.
2. Значение полусуммы огибающих данной функции близко к нулю.

Получение разложения. Процесс отсеивания (sifting)

Residue = s(t)
$I_1(t)$ = Residue
i = 1
k = 1

while Residue not equal zero or not monotone
  while I_i has non-negligible local mean
    U(t) = spline through local maxima of I_i
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    Av(t) = 1/2 (U(t) + L(t))
    $I_i(t) = I_i(t)$ - Av(t)
    i = i + 1
  end
  $IMF_k(t) = I_i(t)$
  Residue = Residue - IMF_k
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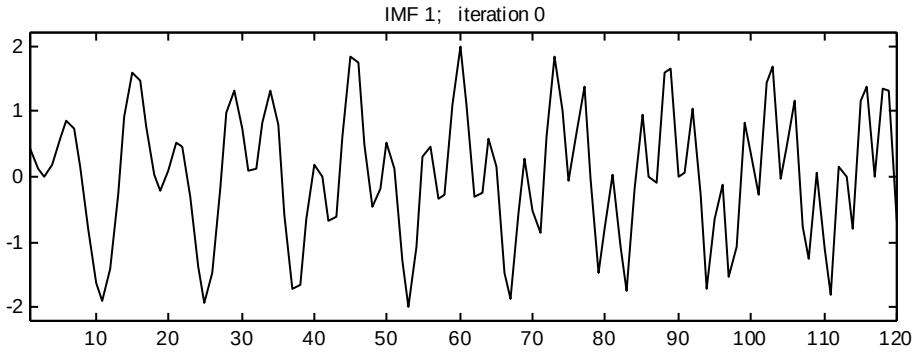

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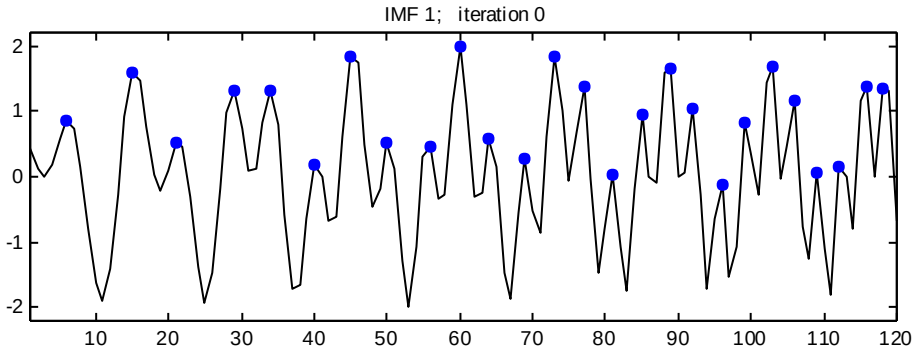

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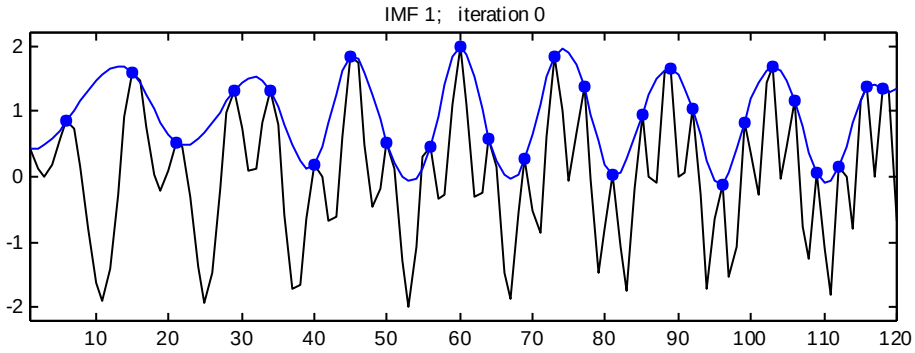

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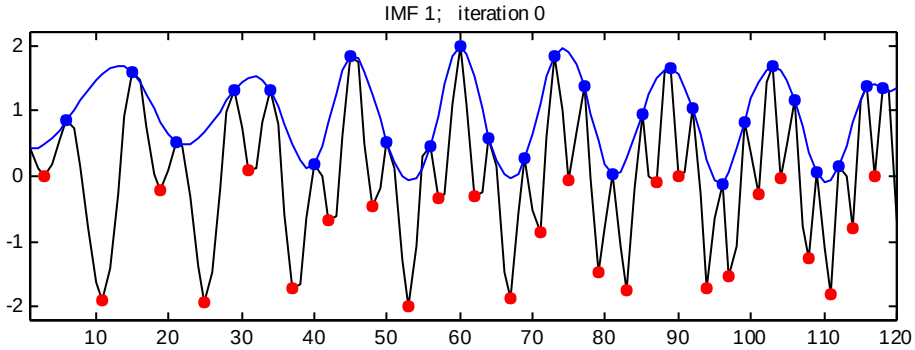

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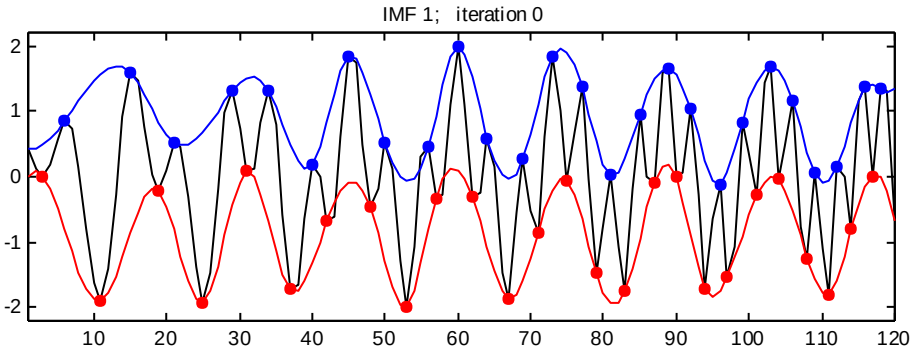

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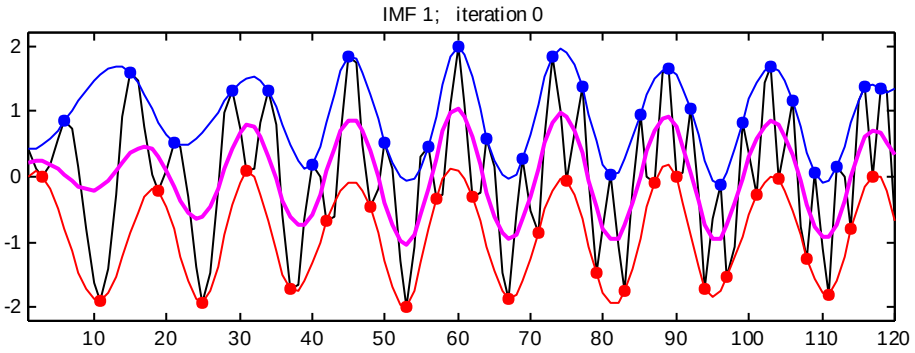

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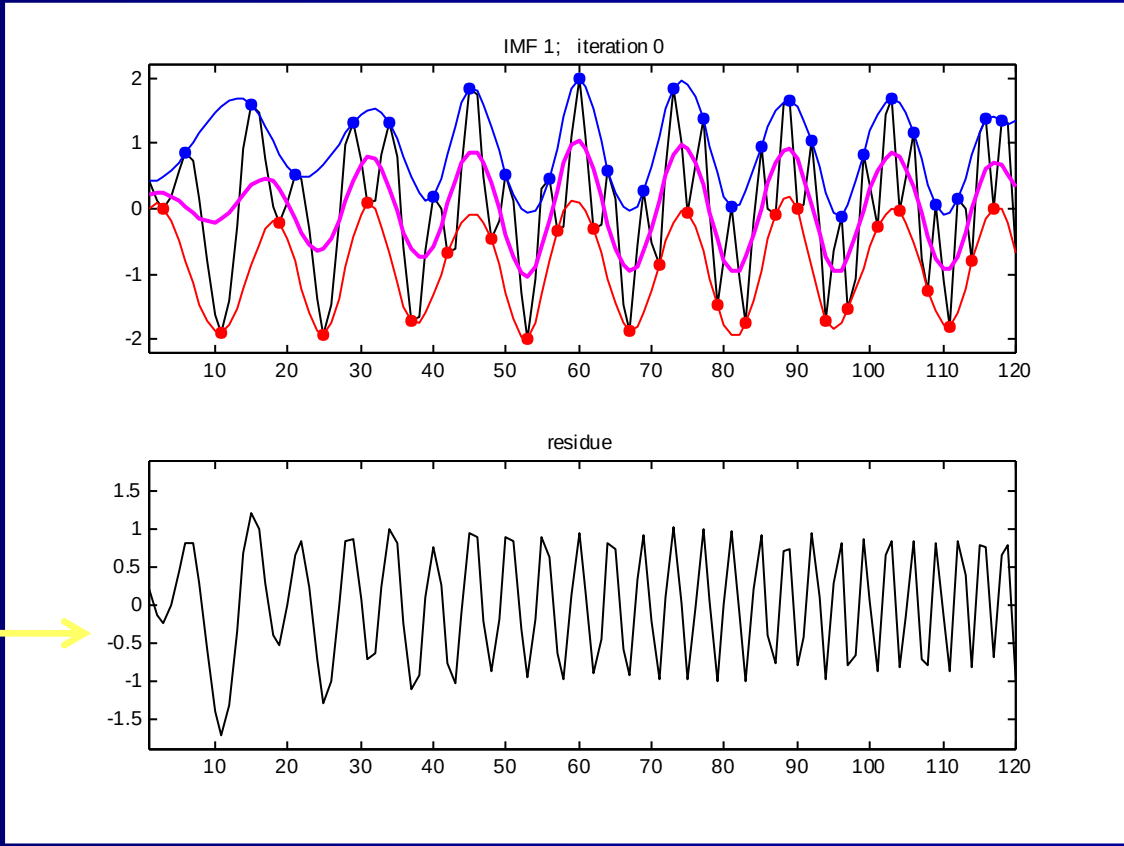

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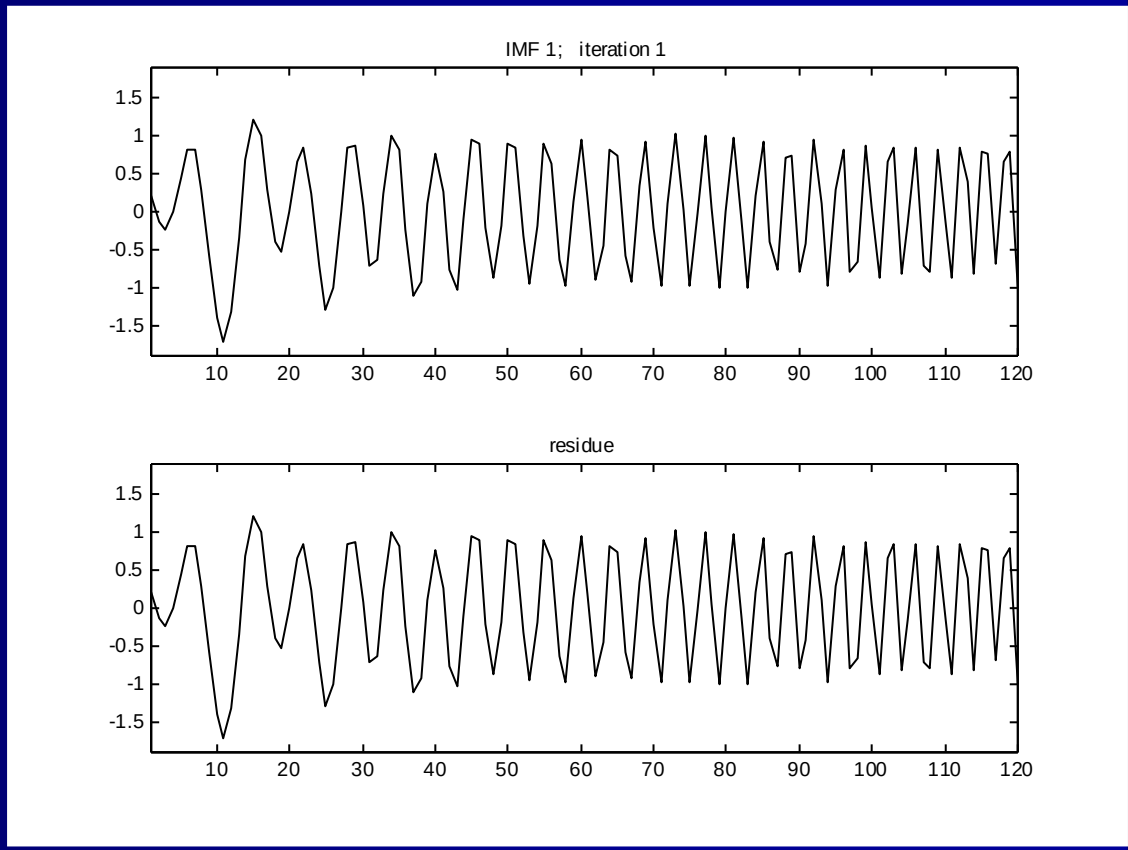

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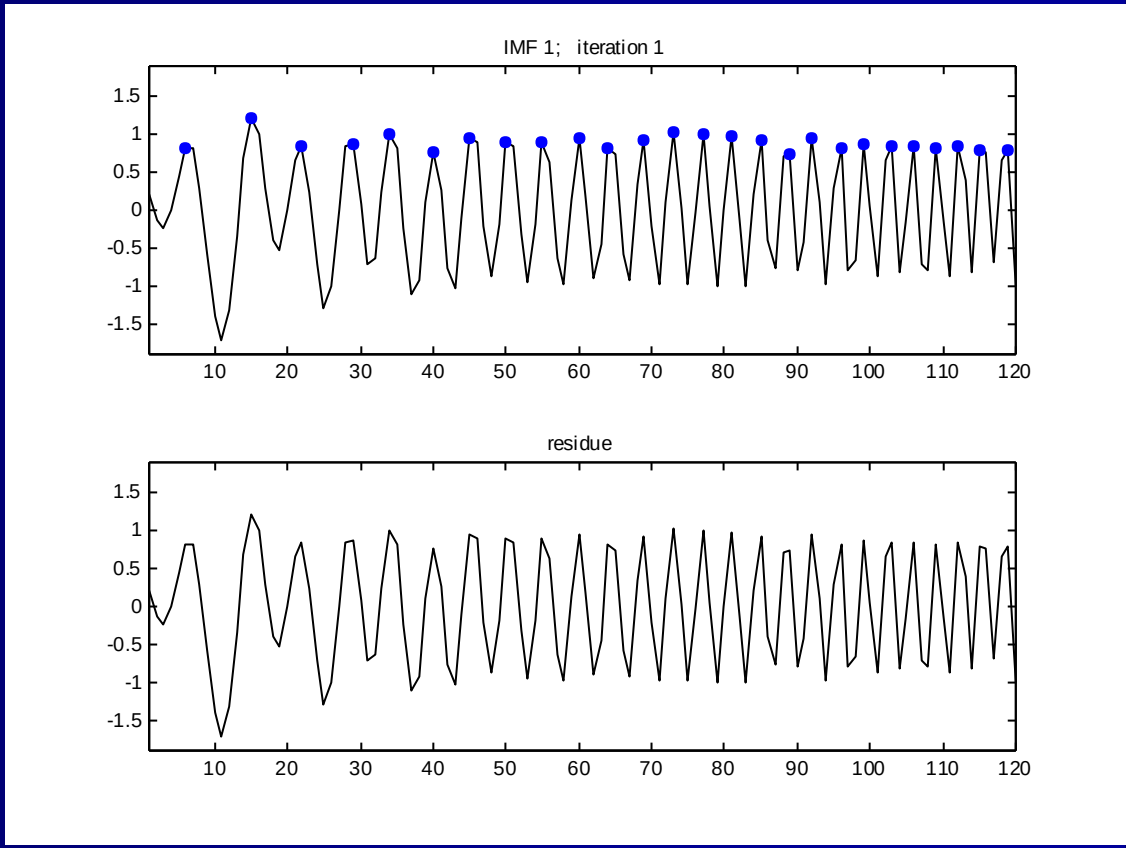

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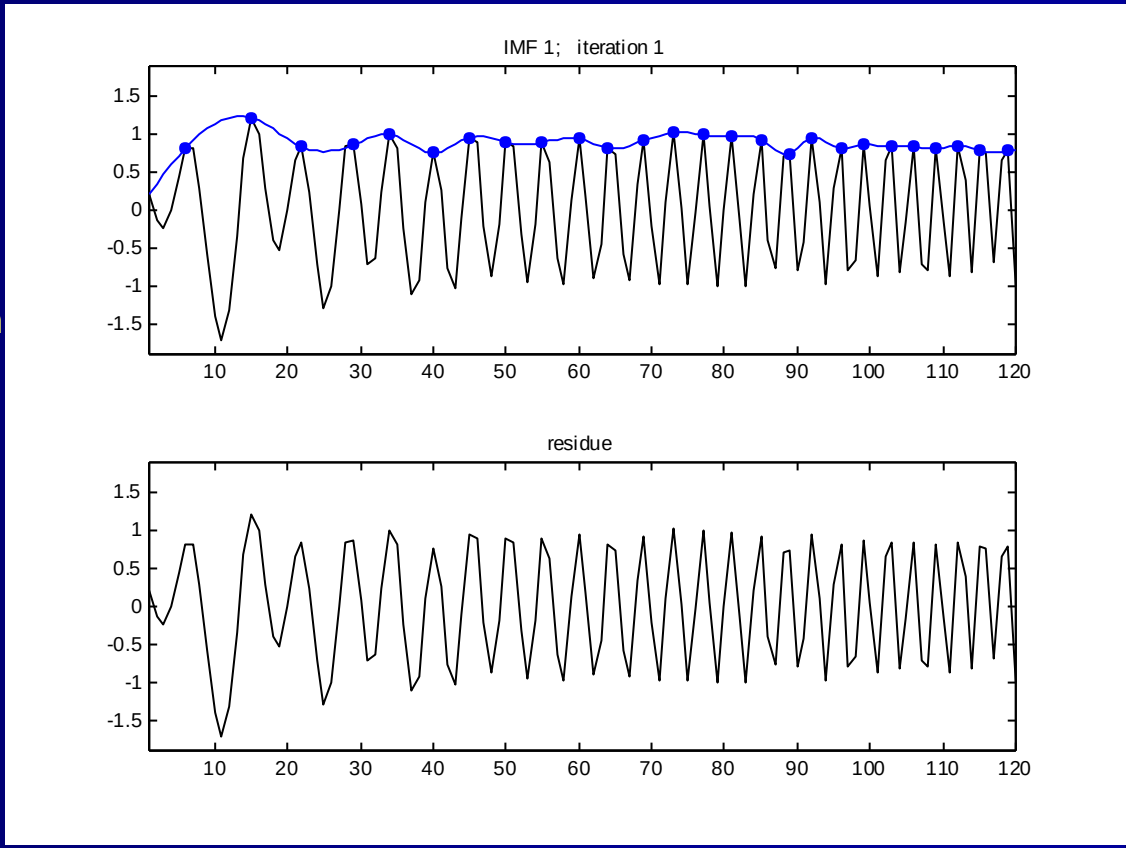

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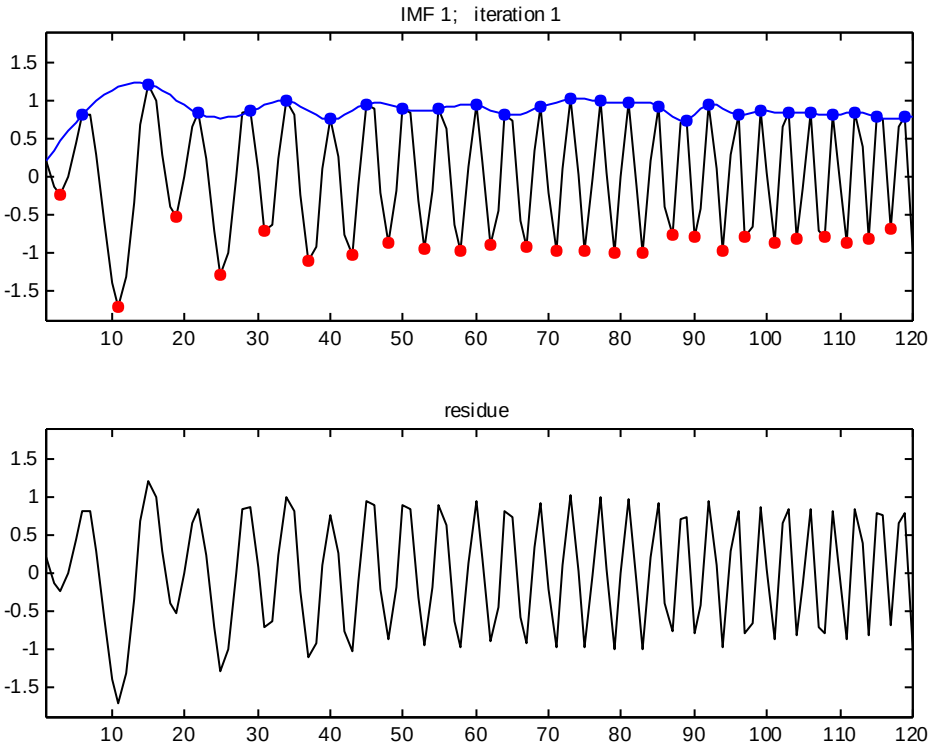

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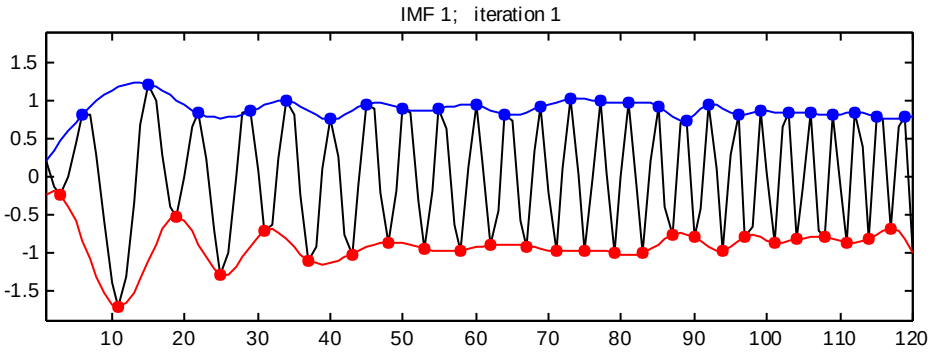

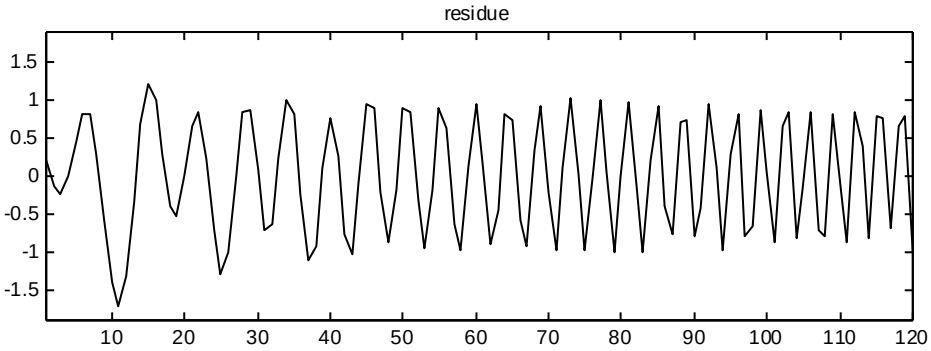

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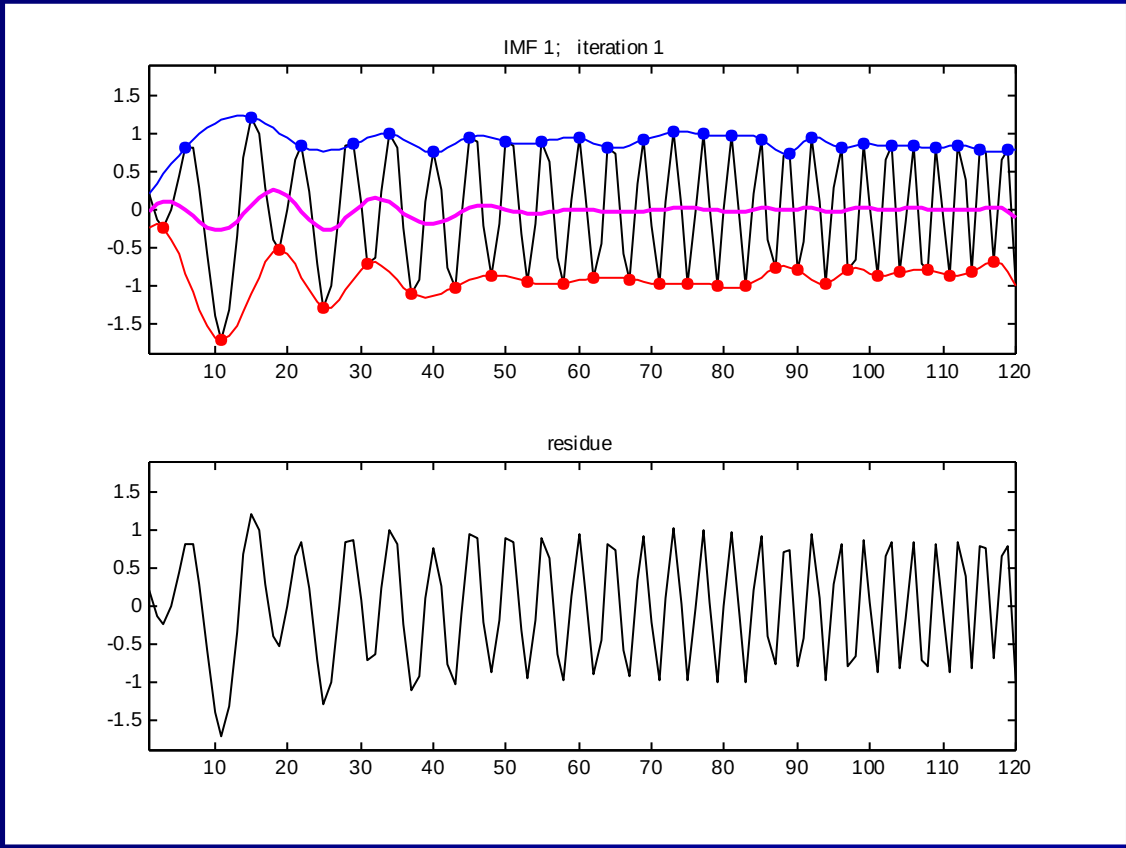

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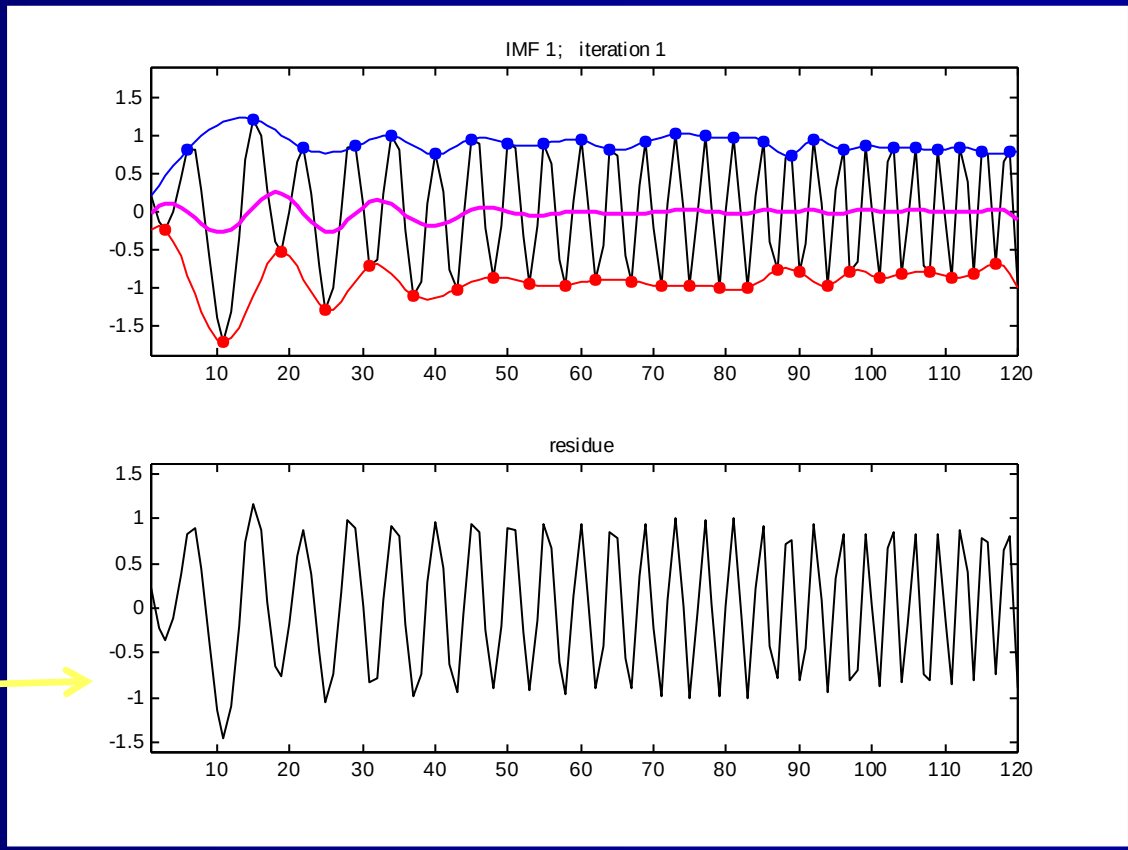

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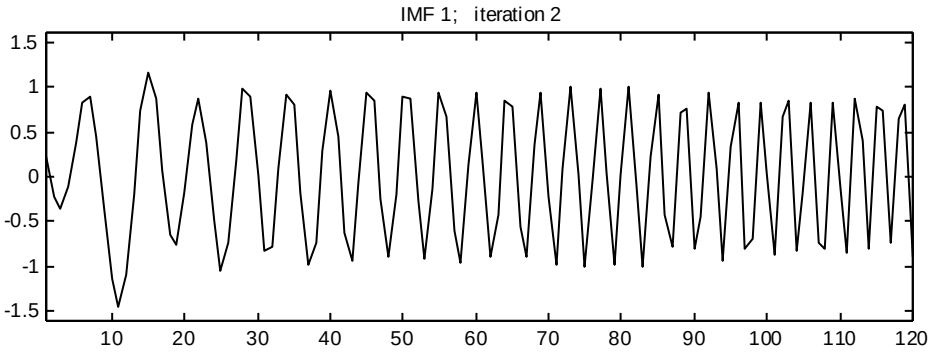

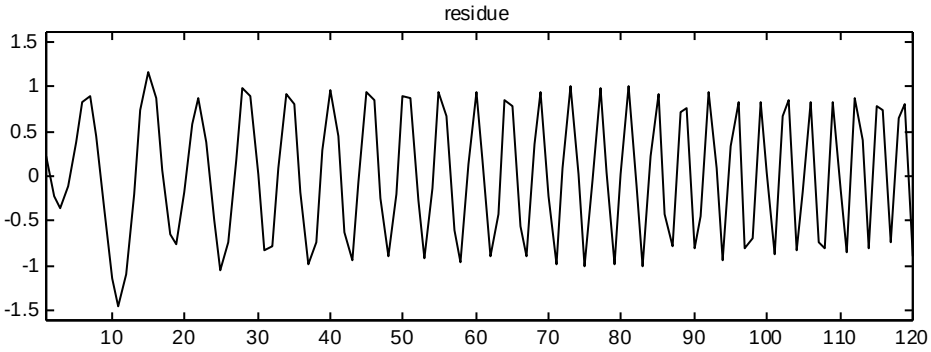

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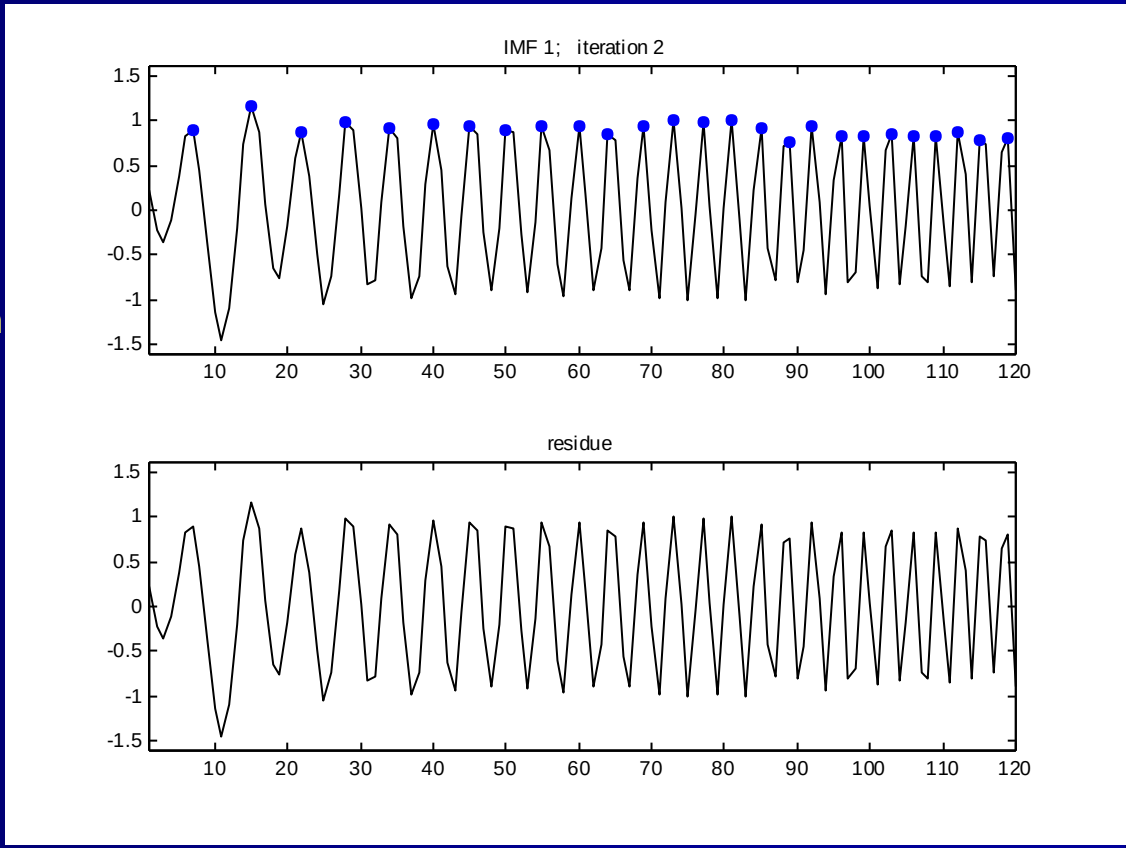

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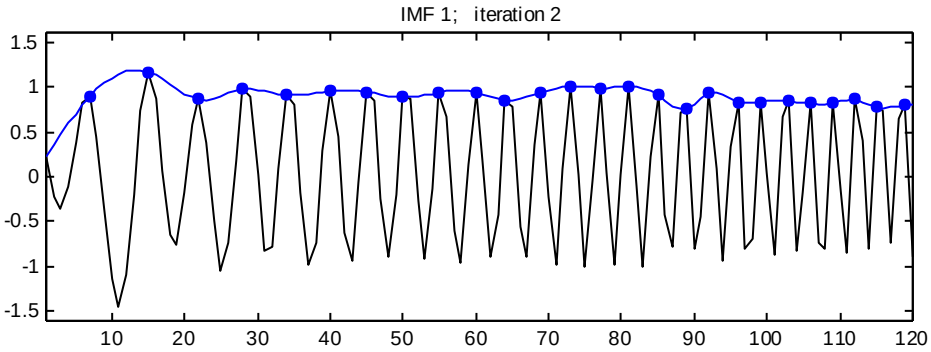

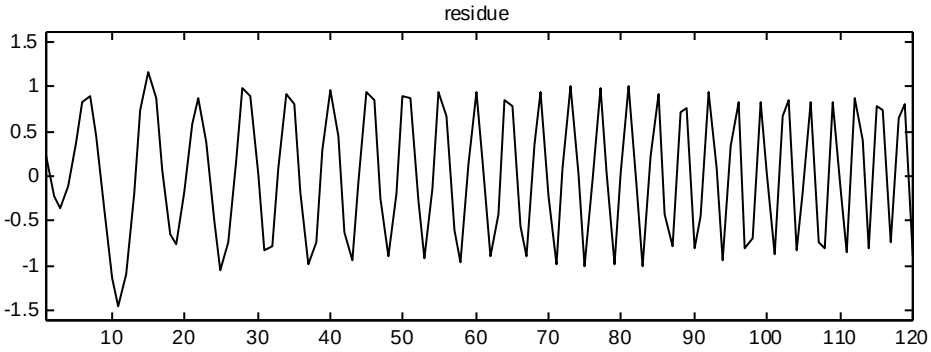

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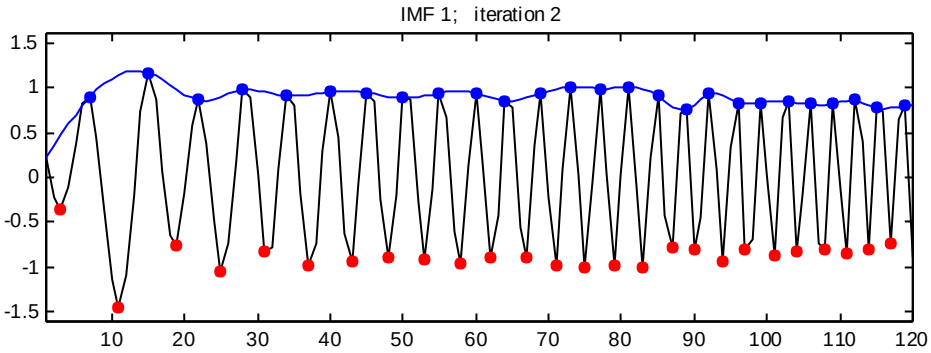

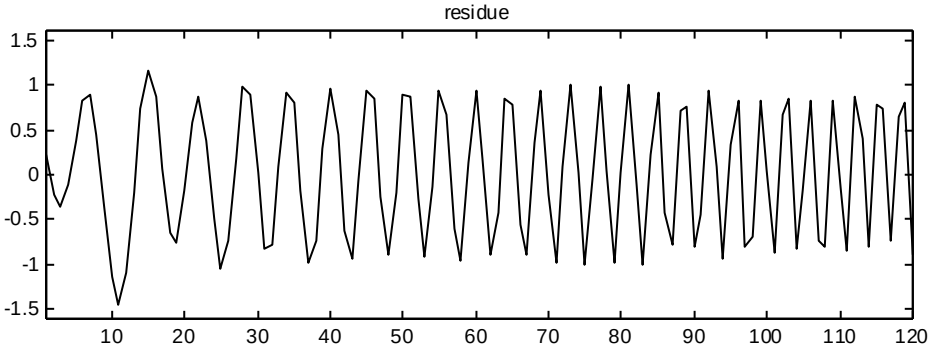

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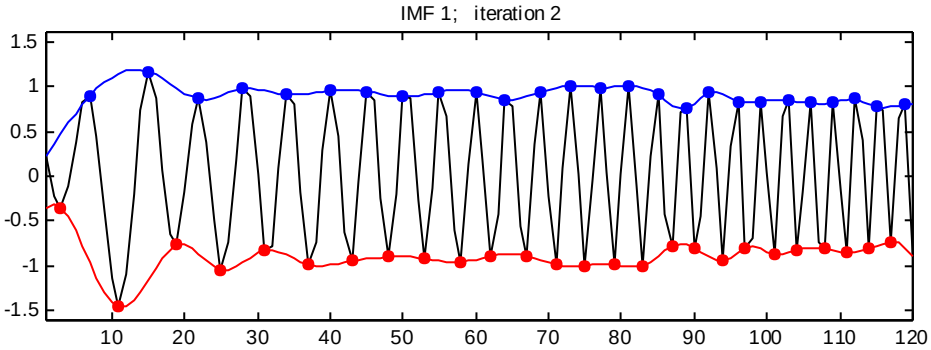

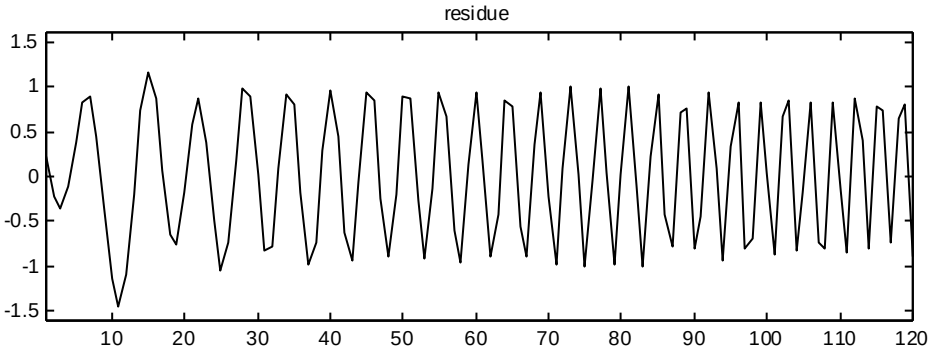

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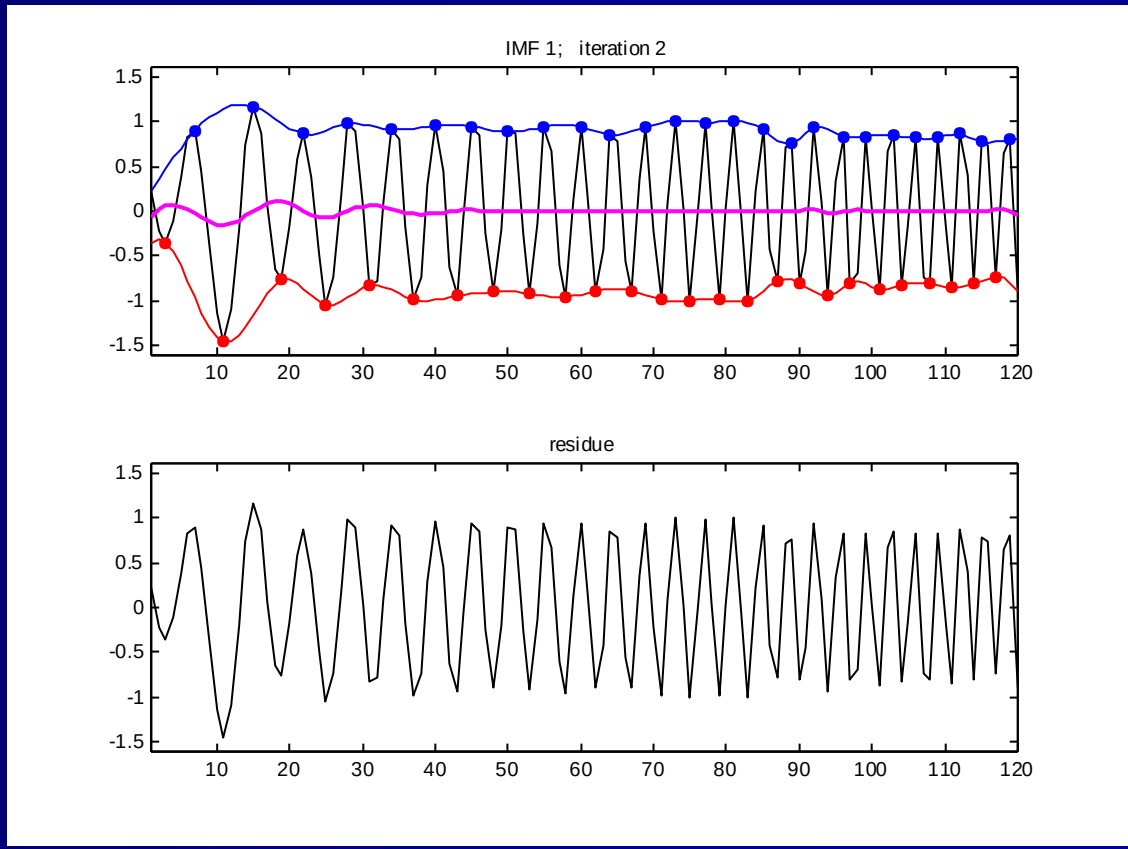

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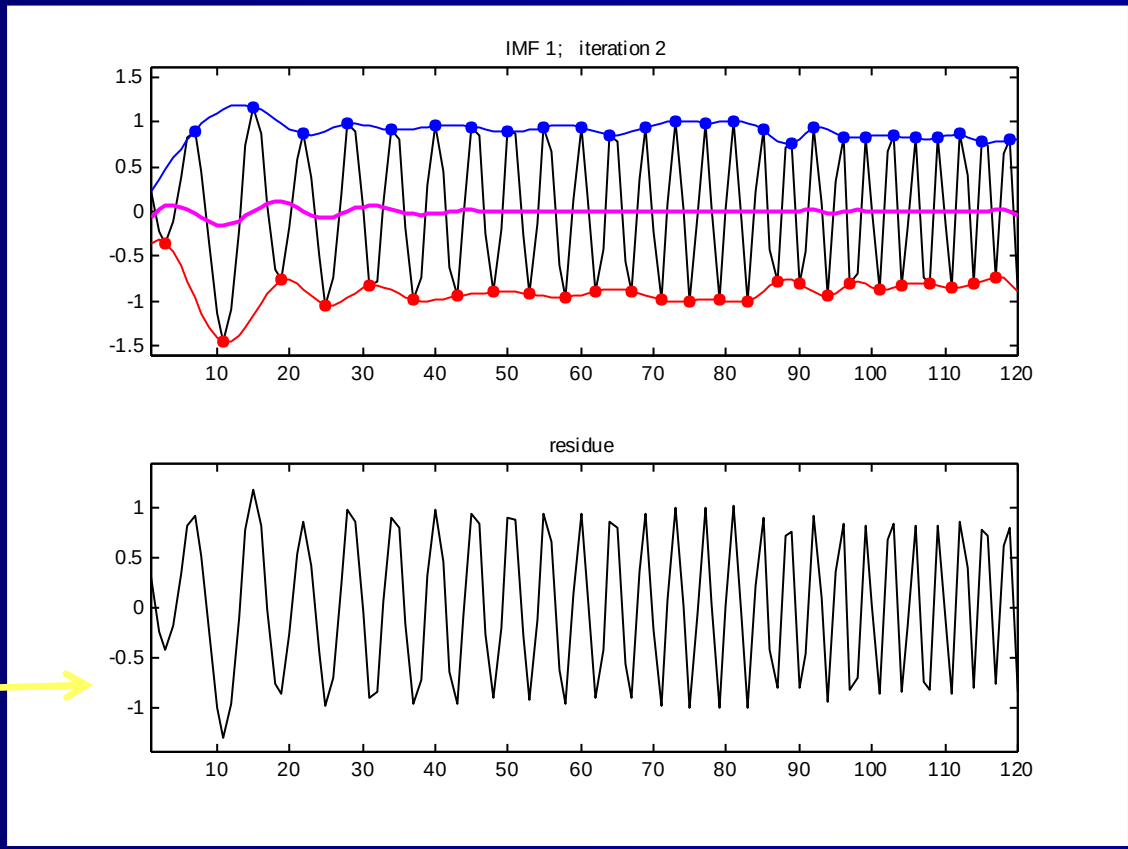

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$IMF_k(t) = I_i(t)$
Residue = Residue - IMF_k
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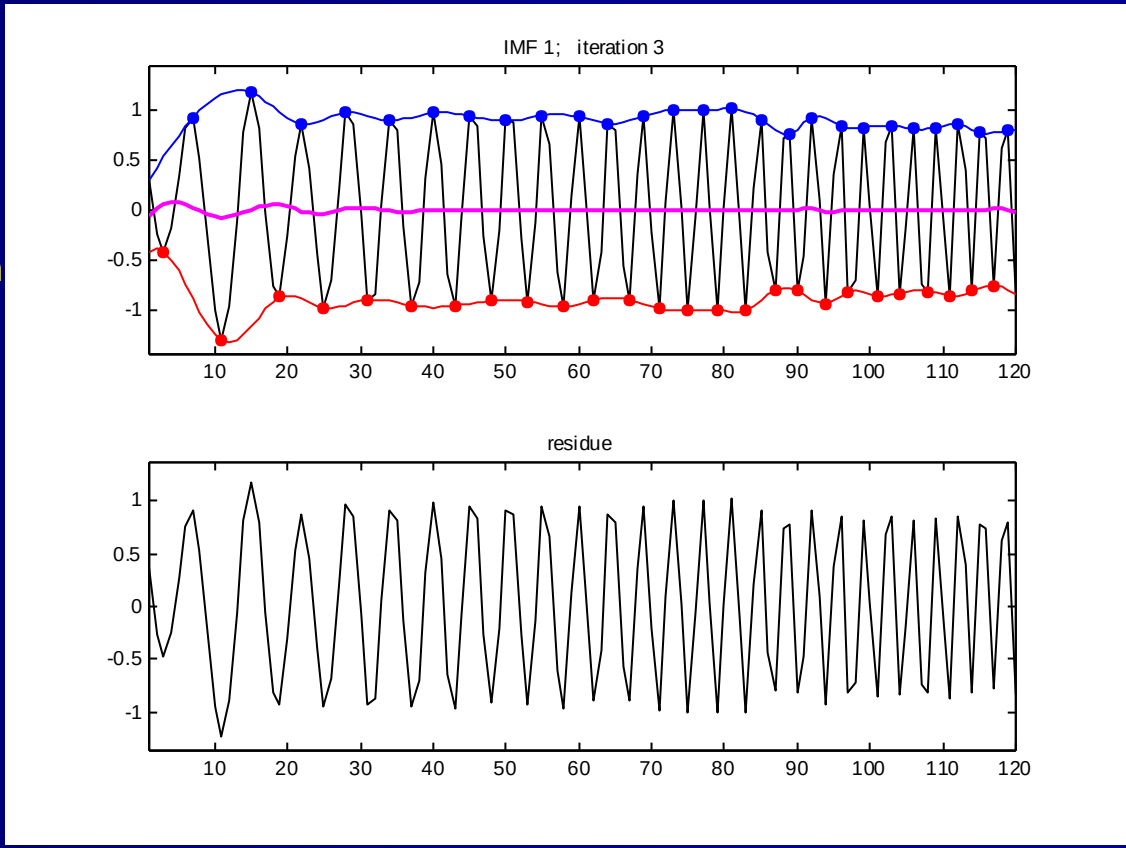
end



Получение разложения. Процесс отсеивания (sifting)

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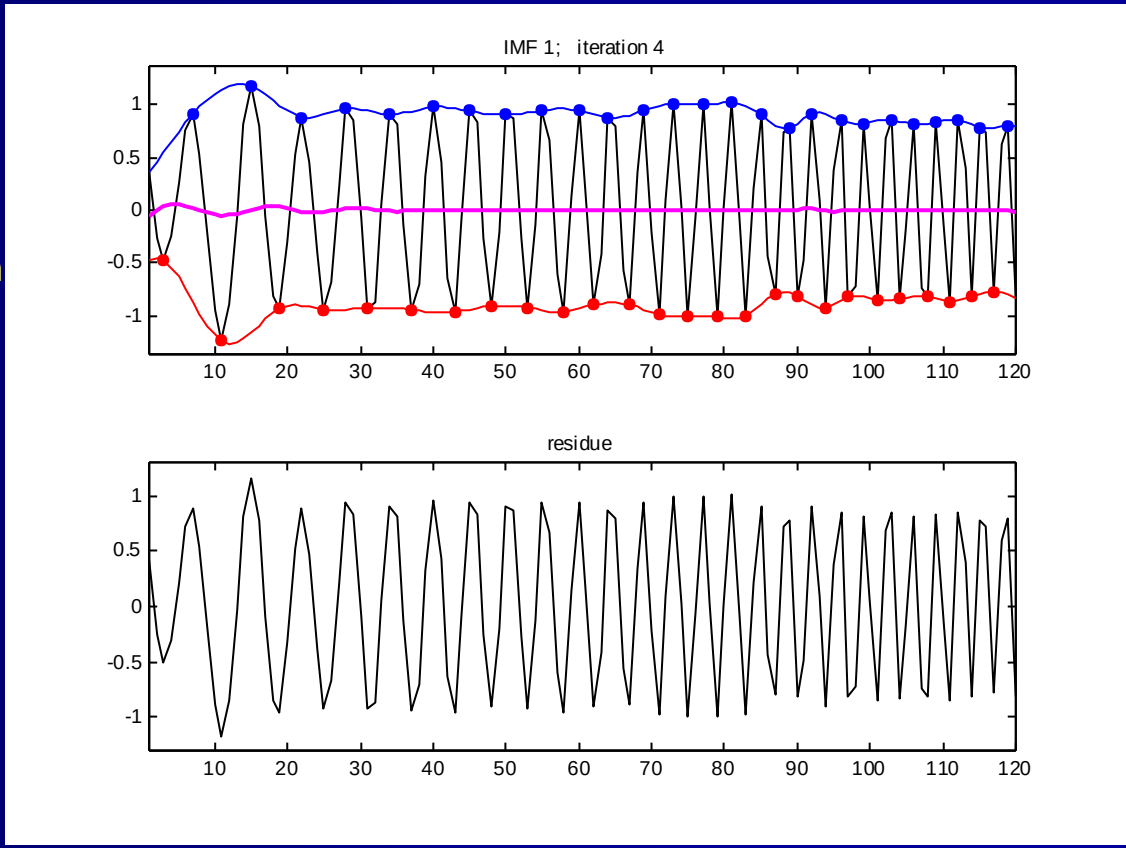

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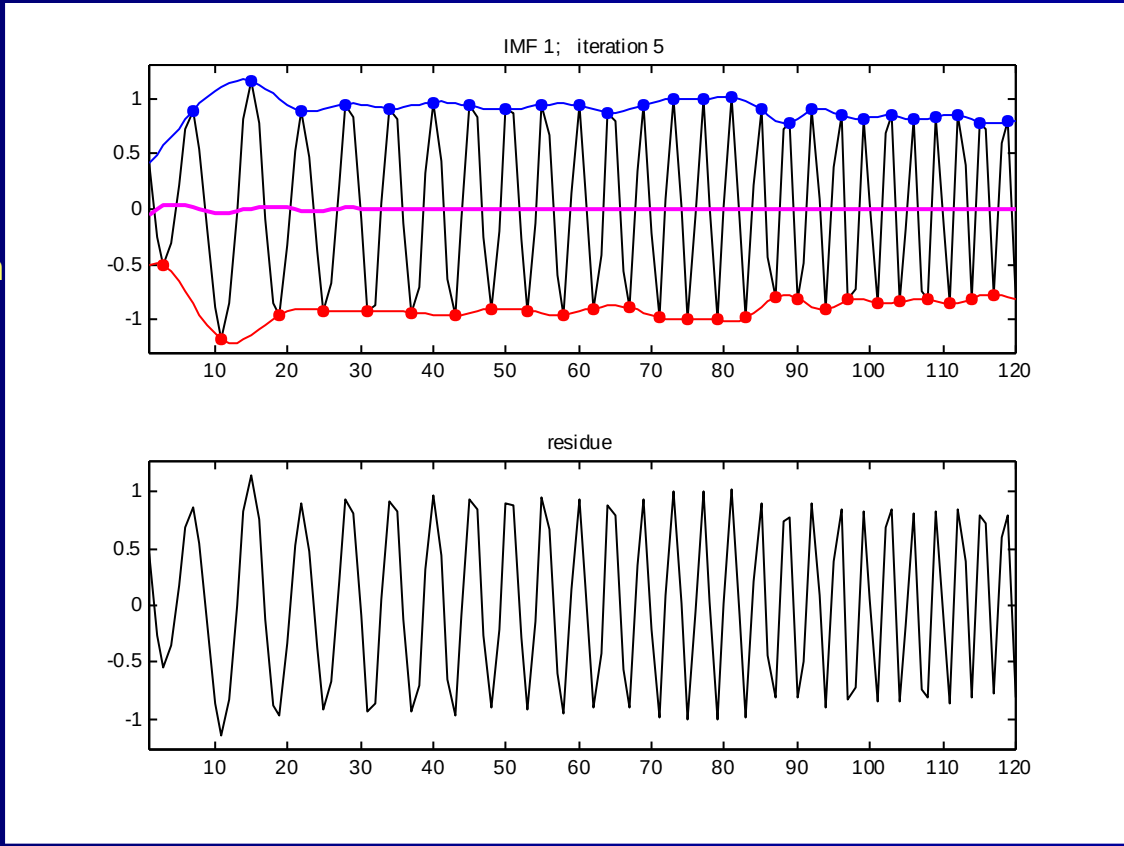

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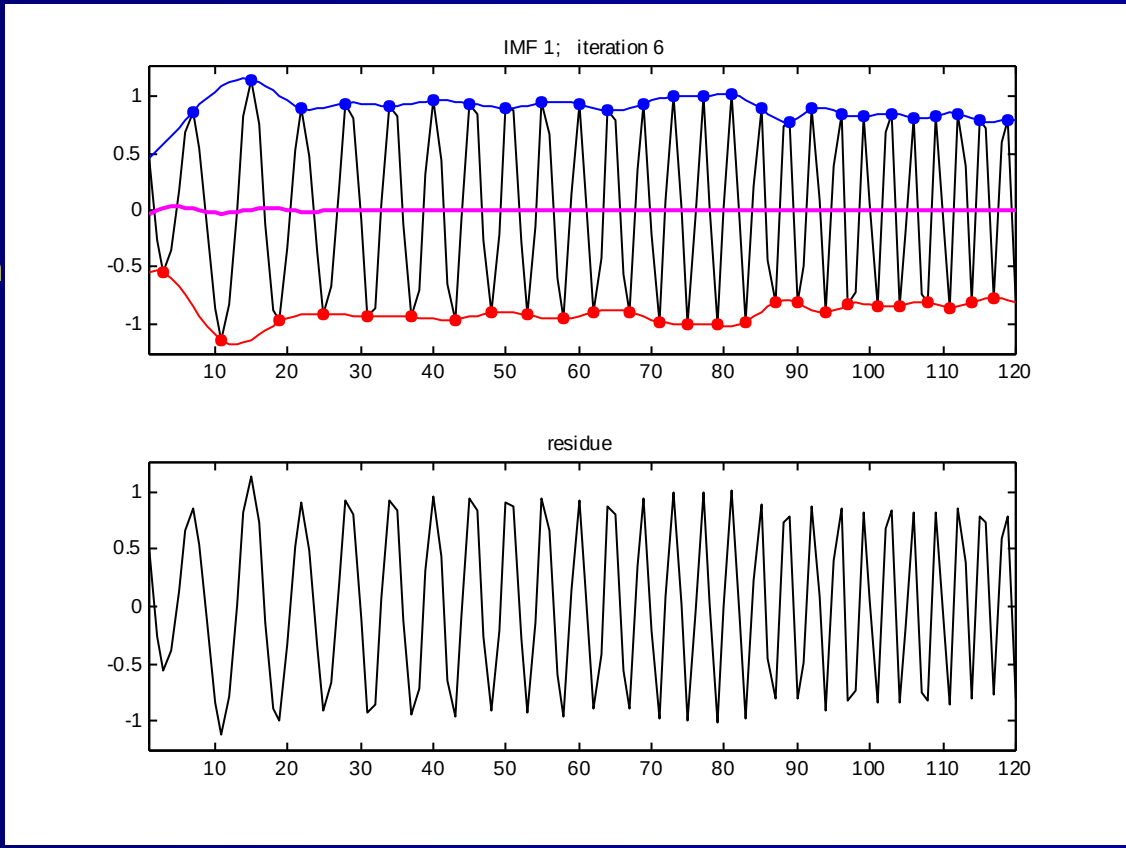

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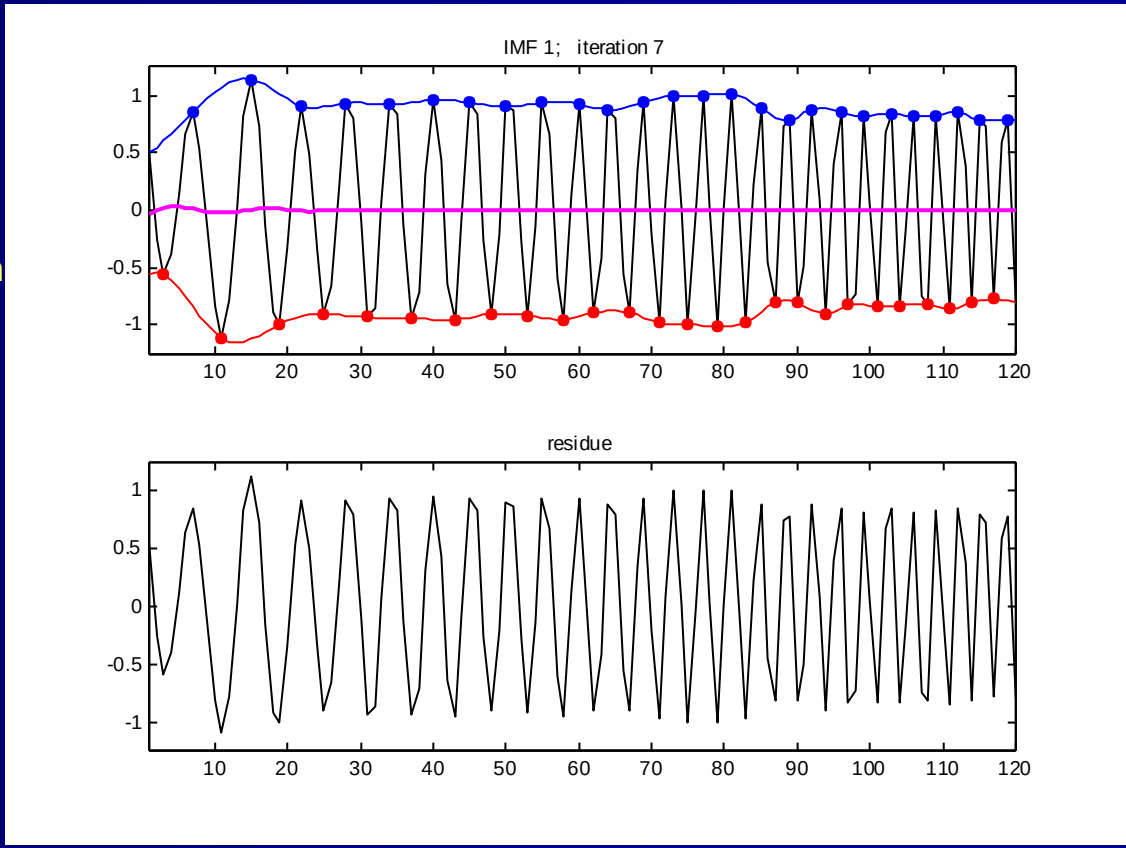

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$IMF_k(t) = I_i(t)$
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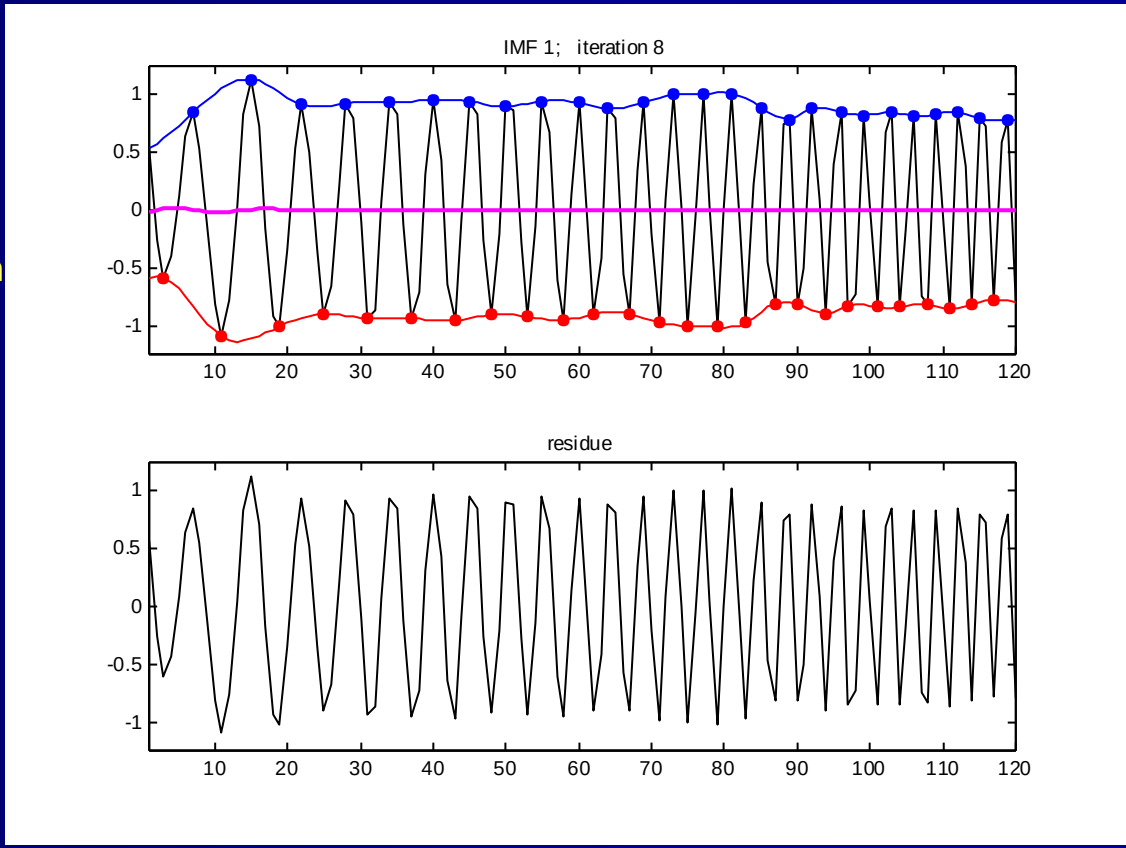
end



Получение разложения. Процесс отсеивания (sifting)

Residue = s(t)
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    while Residue not equal zero or not monotone
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            Av(t) = 1/2 (U(t) + L(t))
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            i = i + 1
        end
        $IMF_k(t) = I_i(t)$
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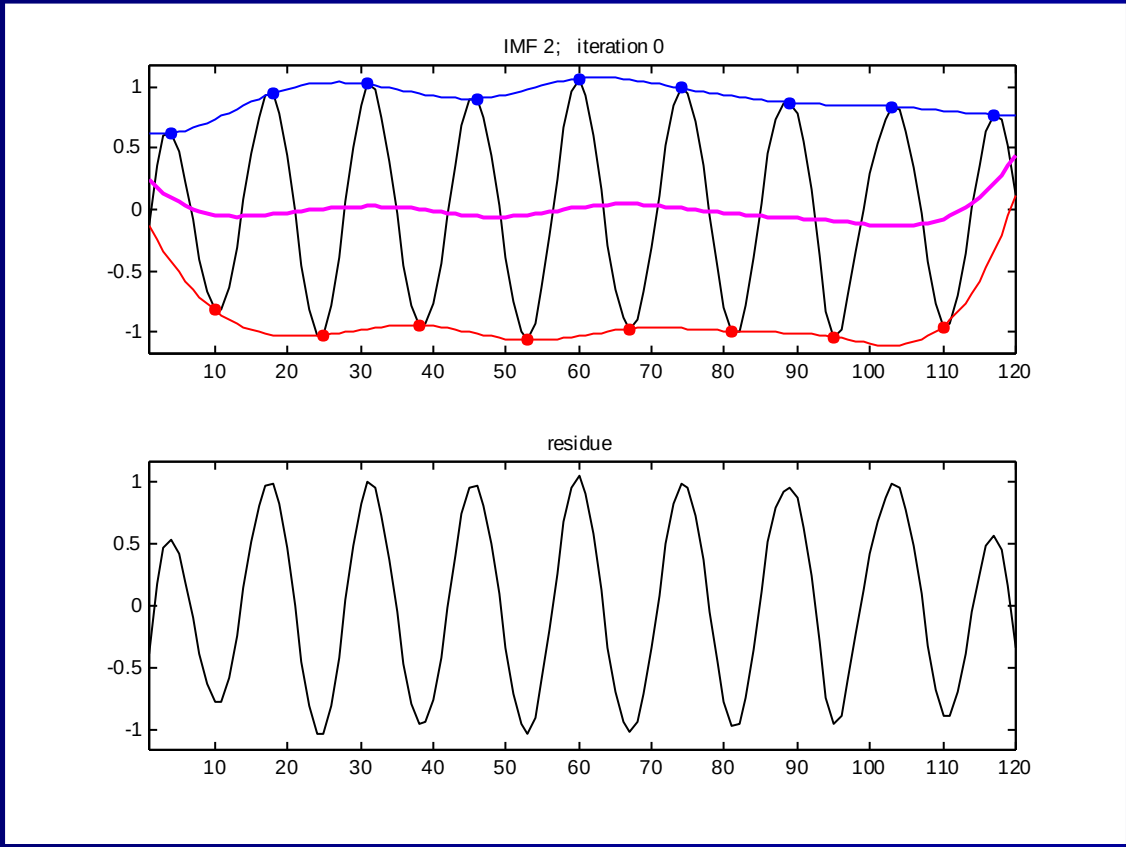
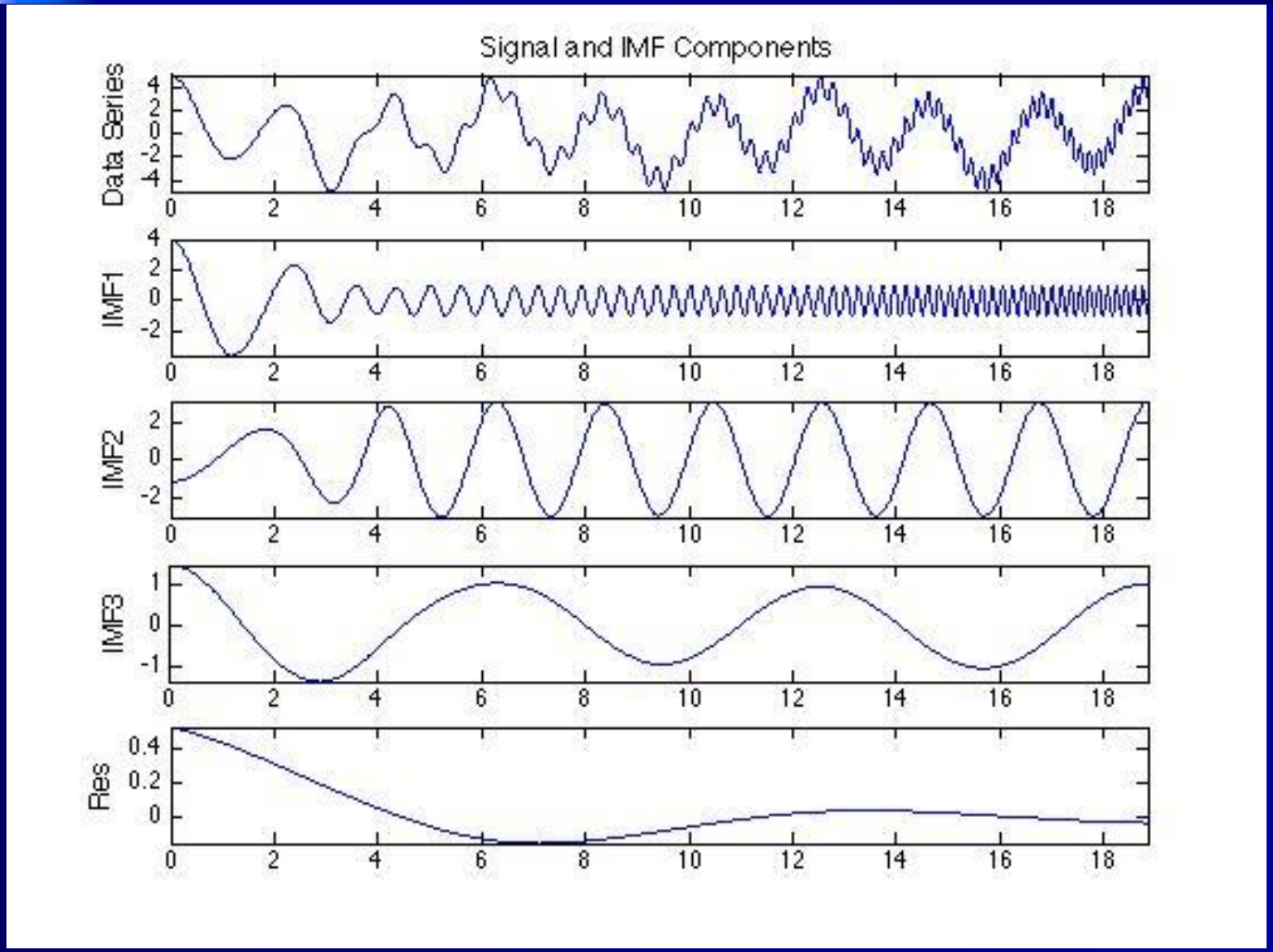
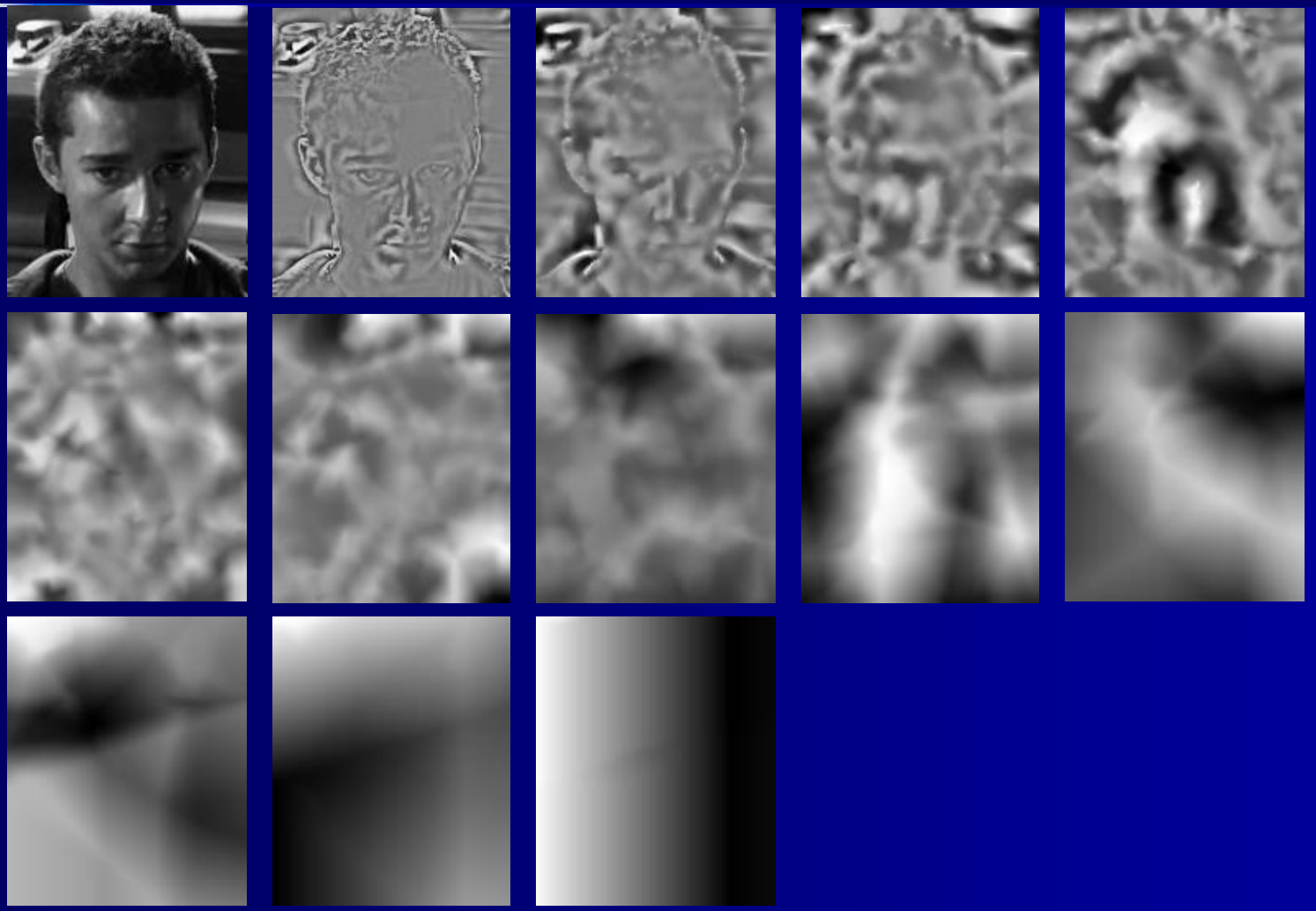
        k = k+1
    end



Результат разложения



EMD для изображений



Коррекция освещённости

Раскладываем исходное изображение с помощью EMD:

$$I(i,j) = \sum_{k=1}^{N} \psi_k(i,j) + r(i,j)$$

Для восстановления результата используем лишь несколько первых IMF:

$$I_{res}(i,j) = \sum_{k=1}^{M} \psi_k(i,j),\ M < N$$

N = 11; M = 9