# Gauss-Laguerre Keypoints Descriptors for Color Images

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***Abstract*—Keypoints descriptors construction algorithm for color images is introduced. The algorithm is based on multiscale Gauss-Laguerre circular harmonic functions expansions of each image color component. The color components are obtained from color invariance model derived from Kubelka-Munk photometric reflectance theory. The matching results of proposed algorithm confirm the importance of color information for keypoints descriptors construction.**

## I. Introduction

The local image feature extraction is an initial step of many complex image processing tasks. The methods of keypoints localization and parametrization are widely presented in literature [1]. There are many approaches to the keypoints detection problem [2], [3], [4]. The problem of keypoints descriptor construction is also widely presented in literature [1], [2], [3]. The target property of the keypoints descriptors is the invariance to a class of projective and photometric transformations. The majority of keypoints extraction methods are based on the analysis of image intensity while color information is also very important. The using of color characteristics allows to achieve better matching rate in the case of photometric transformations caused by variations in illumination and shading.

Color-based descriptors construction methods are also presented in literature [5]. The most popular approach for keypoints descriptors construction uses Gaussian filter and its derivatives like in SIFT method [3]. Papers [6], [7] use color-based methods derived from SIFT approach.

In [8] the keypoints extraction method based on Gauss-Laguerre circular harmonic functions was proposed. The method was constructed for grayscale images. The comparison of its matching rate with SIFT method was presented in [9]. In this paper we present its extension to color images on the basis of color invariants proposed in [10]. The color invariants are derived from the Kubelka-Munk photometric reflectance model. The comparison of color and grayscale Gauss-Laguerre keypoints descriptors is demonstrated in the results section.

## II. Gauss-Laguerre Keypoints Extraction

### A. Gauss-Laguerre Circular Harmonic Functions

Gauss-Laguerre circular harmonic functions (CHFs) form a family of orthonormal and polar separable functions in complex space and defined as:

$$\Psi_n^\alpha(r,\gamma;\sigma) = \psi_n^{|\alpha|}(r^2/\sigma)e^{i\alpha\gamma} \ ,$$

and referenced by integers $n = 0, 1, ...$ (referred by radial order) and $\alpha = 0, \pm1, \pm2...$ (referred by angular order).

The radial profiles of Gauss-Laguerre CHFs are Laguerre functions:

$$\psi_n^\alpha(x) = \frac{1}{\sqrt{n!\Gamma(n+\alpha+1)}} x^{\alpha/2} e^{-x/2} L_n^\alpha(x) \ ,$$

where $L_n^\alpha(x)$ are Laguerre polynomials [9].

Being linearly related to the 2D Hermite functions [11] that are the eigenfunctions of Fourier transformation, Gauss-Laguerre CHFs keep their shape under Fourier transformation. This family of functions is also self-steerable, i.e. they can be rotated by the angle $\theta$ using multiplication by the factor $e^{i\alpha\theta}$.

Due to the orthogonality and completeness of $\Psi_n^\alpha$ family in $L_2$ the observed image $I(x, y)$ can be expanded in the neighborhood of the analysis point $x_0, y_0$ for fixed $\sigma$ in Cartesian system as:

$$I(x+x_0, y+y_0) = \sum_{\alpha=-\infty}^{\infty} \sum_{n=0}^{\infty} g_{\alpha,n}(x_0, y_0; \sigma) \Psi_n^\alpha(\rho, \omega; \sigma) \ ,$$

where

$$\rho = \sqrt{x^2+y^2}, \ \omega = \arctan(\frac{y}{x}) \ ,$$

$$g_{\alpha,n}(x_0, y_0; \sigma) = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} I(x+x_0, y+y_0) \overline{\Psi_n^\alpha(\rho, \omega; \sigma)} dxdy \ .$$

The scale parameter is denoted as $\sigma$ and corresponds to the size of the support region of the given point $x_0, y_0$.

The keypoints detection and descriptors construction is performed on the basis of the $g_{\alpha,n}$ coefficients analysis. Complete description of the keypoints detection algorithm and local descriptors construction process can be found in [8], [9]. In this paper a brief review of these algorithms is presented.

### B. Keypoints Detection

Due to the specific property of Gauss-Laguerre CHFs as natural detectors of different image features (like crosses and corners) [4] the keypoints detection is based on the analysis of

$g_{3,0}$ and $g_{4,0}$ coefficients at different scales. For this purpose the image scalogram is defined:

$$S(x, y; \sigma) = |g_{3,0}(x, y; \sigma)|^2 + |g_{4,0}(x, y; \sigma)|^2, \sigma \in \{\sigma_j\} .$$

It is constructed on a set of different scales $\{\sigma_j\}$. The set of keypoints $\overline{K} = (\overline{x}, \overline{y}; \overline{\sigma})$ is defined as local maxima of the scalogram higher than a selected threshold.

### C. Keypoints Descriptor Construction

Each keypoint $\overline{K}$ is associated with a local descriptor. The descriptor is a complex-valued vector consisted of $g_{\alpha,n}$ for given $n$ and $\alpha$ range at the keypoint reference scale $\overline{\sigma}$ and $2j_{\max}$ scales neighbor to it. The vector is defined as:

$$\overline{\chi}(n, \alpha, j) = \frac{g_{\alpha,n}(x, y; \sigma_j) \cdot e^{-i\alpha\theta_j}}{\|g_{\alpha,n}(x, y; \sigma_j) \cdot e^{-i\alpha\theta_j}\|} , \quad (1)$$

$$n = 0, ..., n_{\max} , \alpha = 1, ..., \alpha_{\max} , j = -j_{\max}, ..., j_{\max} ,$$

where $\sigma_j$ is the j-th scale following $\overline{\sigma}$ if $j > 0$, or preceding $\overline{\sigma}$ if $j < 0$ in the $\{\sigma_j\}$ set of the scales. The normalization makes descriptor stable to the contrast changes. The phase shift $e^{-i\alpha\theta_j}$ is used to make the descriptors stable to the keypoint pattern orientation, where $\theta_j = \arg(g_{1,0}(x, y; \sigma_j))$ .

### D. Acceleration of Descriptor Construction

The descriptors construction process can be accelerated using 2D Hermite functions image expansion. The details of the acceleration procedure is described in [9]. The main idea of acceleration is presented in this paper.

The 2D Hermite functions $\Phi_{m,n}(x, y; \sigma)$ form the complete orthonormal system in $L_2$ space and can be defined as:

$$\Phi_{m,n}(x, y; \sigma) = \frac{1}{\sigma}\phi_m\left(\frac{x}{\sigma}\right)\phi_n\left(\frac{y}{\sigma}\right) ,$$
$$\phi_n(x) = \frac{1}{\sqrt{2^n n! \sqrt{\pi}}} e^{-\frac{x^2}{2}} H_n(x) ,$$

where $n = 0, 1, 2...$ and $H_n(x)$ are Hermite polynomials [9].

The given image $I(x, y)$ can be expanded in the analysis point $x_0, y_0$ for fixed $\sigma$ into a set of 2D Hermite functions $\Phi_{m,n}$:

$$I(x + x_0, y + y_0) = \sum_{m=0}^{\infty}\sum_{n=0}^{\infty} h_{m,n}(x_0, y_0; \sigma)\Phi_{m,n}(x, y; \sigma) ,$$

where

$$h_{m,n}(x_0, y_0; \sigma) = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} I(x+x_0, y+y_0)\Phi_{m,n}(x, y; \sigma)dxdy . \quad (2)$$

As 2D Hermite functions $\Phi_{m,n}$ are linearly related to the Gauss-Laguerre CHFs $\Psi_n^\alpha$ [8], [11], the image expansions coefficients $h_{m,n}$ and $g_{\alpha,n}$ are also linearly related. The separability of $\Phi_{m,n}$ allows to reduce the calculation of the double integral in the right part of (2) to the repeated integral. This accelerates the computation up to 3.5 times [9]. The further acceleration in $\approx 1.5$ times can be achieved using fast Hermite projection method [9], [12].

### E. Color Extension of Keypoints Descriptors

In [8] this Gauss-Laguerre keypoints detection and descriptors construction approach is presented for grayscale images. In this paper the descriptors construction algorithm is extended to color images while the keypoints detection algorithm works with only intensity data. The color Gauss-Laguerre descriptors are constructed concatenating descriptors that were obtained for each color component independently. The procedure of choosing the color components for descriptor construction is described in the next section.

## III. Color Invariance Model

The color invariance model proposed in [10] is used for our approach to keypoints descriptors construction for color images. This model is based on Kubelka-Munk theory of the reflection spectrum of colored bodies. The main equation of this model is

$$E(\lambda, \overrightarrow{x}) = e(\lambda, \overrightarrow{x})(1-\rho_f(\overrightarrow{x}))^2 R_\infty(\lambda, \overrightarrow{x}) + e(\lambda, \overrightarrow{x})\rho_f(\overrightarrow{x}) , \quad (3)$$

where $\lambda$ is the wavelength and $\overrightarrow{x}$ is denotes the position of imaging plane. The illumination spectrum is denoted as $e(\lambda, \overrightarrow{x})$ and $\rho_f(\overrightarrow{x})$ is Fresnel reflectance at $\overrightarrow{x}$. The material reflectivity is denoted as $R_\infty(\lambda, \overrightarrow{x})$ and the reflected spectrum in the viewing direction is represented by $E(\lambda, \overrightarrow{x})$. This model is defined for non transparent objects. Another requirement is high enough resolution of imaging to consider image pixels locally planar.

Assume that the scene is uniformly illuminated with equal spectral energy. Hence the source illumination $e(\lambda, \overrightarrow{x})$ is constant over the wavelengths and varied over the position and can be denoted as $i(\overrightarrow{x})$. So the Eq. (3) is transformed:

$$E(\lambda, \overrightarrow{x}) = i(\overrightarrow{x})\left((1 - \rho_f(\overrightarrow{x}))^2 R_\infty(\lambda, \overrightarrow{x}) + \rho_f(\overrightarrow{x})\right) . \quad (4)$$

Differentiating Eq. (4) with respect to $\lambda$ twice results:

$$E_\lambda = i(\overrightarrow{x})(1 - \rho_f(\overrightarrow{x}))^2 \frac{\partial R_\infty(\lambda, \overrightarrow{x})}{\partial\lambda} \quad (5)$$

and

$$E_{\lambda\lambda} = i(\overrightarrow{x})(1 - \rho_f(\overrightarrow{x}))^2 \frac{\partial^2 R_\infty(\lambda, \overrightarrow{x})}{\partial\lambda^2} . \quad (6)$$

Dividing Eq. (5) by Eq. (6) one can get:

$$H = \frac{E_\lambda}{E_{\lambda\lambda}} = \frac{\partial R_\infty(\lambda, \overrightarrow{x})}{\partial\lambda} / \frac{\partial^2 R_\infty(\lambda, \overrightarrow{x})}{\partial\lambda^2}$$

that depends only on the material reflectivity $R_\infty(\lambda, \overrightarrow{x})$. Thus $H$ component is independent of viewpoint, surface orientation, illumination direction, intensity and Fresnel reflectance coefficient.

In the assumption of Eq. (4) let us consider the case of matte and dull surfaces. So the Fresnel reflectance coefficient $\rho_f(\overrightarrow{x}) \approx 0$ and Eq. (4) is reduced to:

$$E(\lambda, \overrightarrow{x}) = i(\overrightarrow{x})R_\infty(\lambda, \overrightarrow{x}) . \quad (7)$$

So the partial differences of Eq. (7) with respect to $\lambda$ are:

$$E_\lambda = i(\overrightarrow{x})\frac{\partial R_\infty(\lambda, \overrightarrow{x})}{\partial\lambda} , E_{\lambda\lambda} = i(\overrightarrow{x})\frac{\partial^2 R_\infty(\lambda, \overrightarrow{x})}{\partial\lambda^2} .$$

Thus one can get two more components $C_\lambda = E_\lambda / E$ and $C_{\lambda\lambda} = E_{\lambda\lambda}/E$ which depend only on material reflectivity $R_\infty(\lambda, \overrightarrow{x})$ and are independent of viewpoint, surface orientation, illumination direction and intensity.

There are some more invariants can be derived from Kubelka-Munk model with some other assumptions by spatial differentiation. The complete set of the invariants and their properties can be found in [10].

Spectral differential quotients $(\hat{E}, \hat{E}_\lambda, \hat{E}_{\lambda\lambda})$ is used as $(E, E_\lambda, E_{\lambda\lambda})$ components. Spectral differential quotients $(\hat{E}, \hat{E}_\lambda, \hat{E}_{\lambda\lambda})$ are obtained from known RGB components using the Gaussian color model. The $\hat{E}$, $\hat{E}_\lambda$ and $\hat{E}_{\lambda\lambda}$ components of the Gaussian color model well approximate the CIE 1964 XYZ basis linearly for $\lambda_0 = 520\text{nm}$ and $\sigma_\lambda = 55\text{nm}$ [10]. Meanwhile XYZ basis is linearly related to RGB components. Thus one can obtain the $\hat{E}$, $\hat{E}_\lambda$ and $\hat{E}_{\lambda\lambda}$ components from RGB image components using the following formula [10]:

$$\begin{bmatrix} \hat{E} \\ \hat{E}_\lambda \\ \hat{E}_{\lambda\lambda} \end{bmatrix} = \begin{pmatrix} 0.06 & 0.63 & 0.27 \\ 0.3 & 0.04 & -0.35 \\ 0.34 & -0.6 & 0.17 \end{pmatrix} \begin{bmatrix} R \\ G \\ B \end{bmatrix} .$$

All the invariants in [10] that requires spatial differentiation of $E$, $E_\lambda$, $E_{\lambda\lambda}$, $C_\lambda$, $C_{\lambda\lambda}$ and $H$ are obtained using convolution with Gaussian derivatives instead of differentiating. The Gauss-Laguerre approach to descriptors construction [8], [9] is based on image extention into a set of Gauss-Laguerre CHFs. But Gauss-Laguerre CHFs are linearly related to the 2D Hermite functions that are the higher order derivatives of Gaussian. Hence using the $E$, $E_\lambda$, $E_{\lambda\lambda}$, $C_\lambda$, $C_{\lambda\lambda}$ and $H$ color invariants as the initial image in Gauss-Laguerre method allows to obtain higher order invariants as the components of keypoints descriptors.

## IV. Results

The precision-recall characteristics of proposed algorithm were evaluated on the images from the dataset freely available on the web, which provides the images and the relating homographies sequences (http://www.robots.ox.ac.uk/~vgg/research/affine/). The keypoints detector parameters were set for each image pair independently to get about 1000 keypoints per image. Only the color images from this dataset were taken. To demonstrate the performance of the proposed descriptors in the case of illumination changes the images from ALOI database [13] were taken.

The keypoints detection was performed using just the $E$ intensity information. There is a possibility to improve the detection process using color information but in general case the improvement looks to have slight influence on the matching results. Although in some special cases of imaging it looks possible to get significant improvement in matching. So this is the question of future investigation.

The comparison of different combinations of color invariants for descriptor construction was performed. The values of parameters $n_{\max}$, $\alpha_{\max}$ and $j_{\max}$ were also varied to achieve the optimal balance between the resulting vector size and the matching performance. The intensity component $E$ was included into every combination as the most informative component in the general case. The grayscale Gauss-Laguerre descriptors were compared with SIFT descriptors for the values $n_{\max} = 5$, $\alpha_{\max} = 5$ and $j_{\max} = 2$ (denoted as Gauss-Laguerre 5,5,2). It gives the 150 elements in the Gauss-Laguerre descriptor (see Eq. (1)). Hence the same values were used as initial for $E$ intensity channel.

Different combinations of color invariants for descriptors construction were tested: $(E, E_\lambda, E_{\lambda\lambda})$, $(E, H)$, $(E, C_\lambda, C_{\lambda\lambda})$, $(E, E_\lambda, E_{\lambda\lambda}, H)$ and $(E, C_\lambda, C_{\lambda\lambda}, H)$. For $E_\lambda, E_{\lambda\lambda}, C_\lambda, C_{\lambda\lambda}, H$ the $n_{\max}$ and $\alpha_{\max}$ were varied from 2 to 5, $j_{\max}$ was varied from 0 to 2. The most appropriate results were obtained using the combination $(E, C_\lambda, C_{\lambda\lambda})$. The $H$ component does not justify the hopes of being very informative. Moreover the test showed that precision-recall curves changed insignificantly while varying parameters $n_{\max}$, $\alpha_{\max}$ and $j_{\max}$ for $H$ channel. So it is reasonable to use just 1 or 2 coefficients of $H$ channel as additional to the main components of descriptors to increase precision slightly in the case of very high recall requirement.

The $(E, C_\lambda, C_{\lambda\lambda})$ combination with $n_{\max} = 5$, $\alpha_{\max} = 5$ and $j_{\max} = 2$ for all components (denoted as color Gauss-Laguerre 5,5,2-5,5,2) gives the closest curve to the grayscale descriptors results for the low values recall and begins to overcome grayscale results from $\approx 0.42$ recall value. But in this case the initial size of the descriptor triples.

To hold the dimension of the descriptors within appropriate range of about 150 elements the values of $n_{\max}$, $\alpha_{\max}$ parameters for $E$ component were reduced. Tests showed that reduction of scaling parameter $j_{\max}$ leads to decrease in matching performance so only $n_{\max}$ and $\alpha_{\max}$ were varied. The best matching performance was achieved using $n_{\max} = 4$, $\alpha_{\max} = 4$, $j_{\max} = 2$ for $E$ component (100 elements) and $n_{\max} = 2$, $\alpha_{\max} = 2$, $j_{\max} = 2$ for $C_\lambda$, $C_{\lambda\lambda}$ components ($2 \times 30$ elements). This combination is denoted as color Gauss-Laguerre 4,4,2-2,2,2. Thus the overall size of the descriptors is 160 elements that is close to the grayscale analog while the precision-recall characteristic of those descriptors is similar to 450 elements descriptors mentioned above. Even more, this type of descriptors allows to achieve greater value of recall overall.

The comparison of precision-recall graphs of defined above keypoints descriptors is illustrated in Fig. 1.

The precision-recall graphs for proposed descriptors with fast Hermite projection method acceleration [12], [9] are illustrated in Fig. 2.

In Fig. 3 the example of keypoints matching for a pair of images from ALOI database [13] is shown. The overall number of matches for grayscale descriptors is 41 where 4 matches are incorrect. The overall number of matches for color descriptors is 77 where 3 matches are incorrect. The parameters of descriptors matching were the same for both descriptors types.

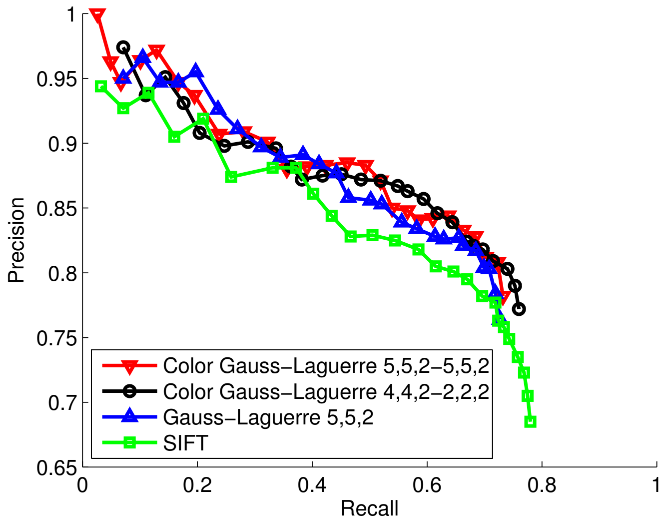


Fig. 1. Precision-recall graphs for graf1-graf2 image pair.

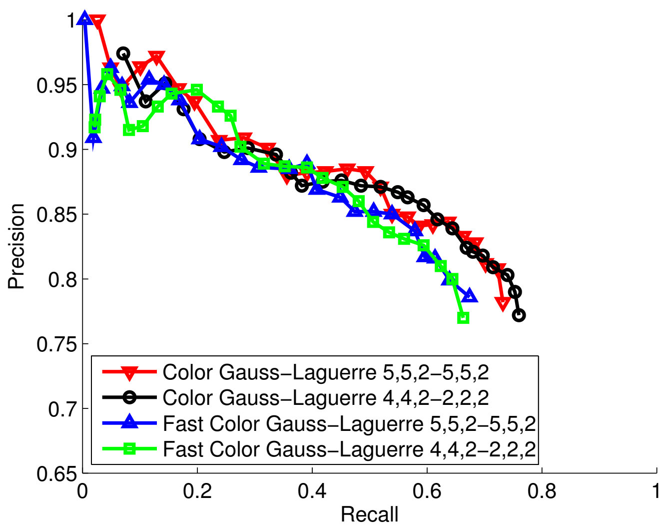


Fig. 2. Precision-recall graphs for graf1-graf2 image pair. Comparison of color Gauss-Laguerre descriptors with fast color Gauss-Laguerre descriptors.

## V. Conclusion

Keypoints descriptors construction algorithm for color images was introduced. The algorithm is based on Gauss-Laguerre CHFs expansion approach [4], [8], [9]. The matching results of proposed algorithm confirm the importance of color information in matching procedure. The presented comparison with Gauss-Laguerre grayscale descriptors and SIFT algorithm looks promising. The using of color information in keypoints detection process and a comparison with existing descriptors for color images is the question of future investigation.

## Acknowledgment

The work was supported by RFBR grant 09-07-92000-HHC.

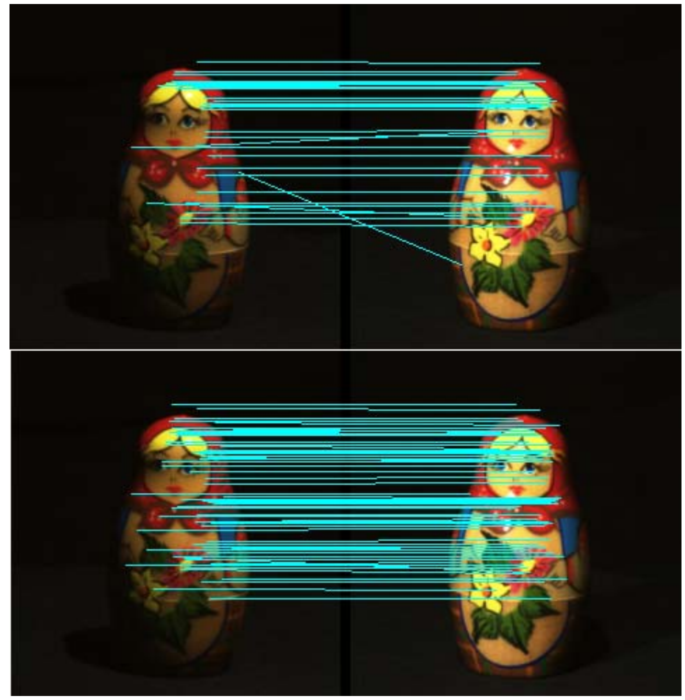

Fig. 3. Example of keypoints matching in the case of illumination changes. The upper image pair was produced using Gauss-Laguerre 5,5,2. The lower image pair was performed using color Gauss-Laguerre 4,4,2-2,2,2.

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