Gauss-Laguerre Keypoints Extraction Using Fast Hermite Projection Method

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Abstract. Keypoints detection and descriptors construction method based on multiscale Laguerre-Gauss circular harmonic functions expansions is considered. Its efficient acceleration procedure is introduced. Two acceleration ideas are used. The first idea is based on the interconnection between Laguerre-Gauss circular harmonic functions system and 2D Hermite functions system. The further acceleration is based on the original fast Hermite projection method. The comparison tests with SIFT algorithm were performed. The proposed method can be additionally enhanced and optimized. Nevertheless even preliminary investigation showed promising results.

Keywords: keypoints extraction, Gauss-Laguerre circular harmonic functions, Hermite functions, fast Hermite projection method, image matching

1 Introduction

The images keypoints extraction is one of the basic problems of low level image processing. Keypoints detection and parametrization is the initial step in tasks like stereo matching [1], object recognition [2], video indexing [3], panorama building and others. There are many approaches to the keypoints detection problem such as Harris corner detector [4], DoG approach presented by Lowe [5], the approach based on circular harmonic functions theory [6], [7], etc. The problem of keypoints descriptor construction is also widely presented in literature [4], [5], [8]. The invariance to a class of projective and photometric transformations is the target property of the descriptor construction algorithm. This property is crucial to obtain high matching rate across multiple views. As the majority of keypoints descriptors construction algorithms are computationally expensive, development of efficient computation algorithms becomes actual.

In this paper the keypoints detection and descriptors construction multiscale approach based on Gauss-Laguerre circular harmonic functions [6] is considered. The 2D Hermite projection-based fast algorithm for efficient exact keypoints descriptors computation is proposed. The structure of the paper is the following: the first section is devoted to the Gauss-Laguerre keypoints extraction and its Hermite projection method acceleration, in the second section the fast Hermite projection method is considered and test results are described in the last section.

2 Gauss-Laguerre Keypoints

2.1 Gauss-Laguerre keypoints detection

Let us consider a family of complex orthonormal and polar separable functions:

$$\Psi(r,\gamma;\sigma) = \psi_n^{|\alpha|}(r^2/\sigma)e^{i\alpha\gamma} .$$

Their radial profiles are Laguerre functions:

$$\psi_n^{|\alpha|}(x) = \frac{1}{\sqrt{n!\Gamma(n+\alpha+1)}} x^{\alpha/2} e^{-x/2} L_n^{\alpha}(x) ,$$

where $n = 0, 1, ...; \alpha = 0, \pm1, \pm2...$ and $L_n^{\alpha}(x)$ are Laguerre polynomials:

$$L_n^{\alpha}(x) = (-1)^n x^{-\alpha} e^x \frac{d}{dx^n}(x^{n+\alpha}e^{-x}) .$$

The Laguerre functions $\psi_n^{\alpha}(x)$ can be calculated using the following recurrence relations:

$$\psi_{n+1}^{\alpha}(x) = \frac{(x-\alpha-2n-1}{\sqrt{(n+1)(n+\alpha+1)}}\psi_n^{\alpha}(x) -$$
$$\sqrt{\frac{n(n+\alpha)}{(n+1)(n+\alpha+1)}}\psi_{n-1}^{\alpha}(x) \ , \ n = 0, 1, ...,$$
$$\psi_0^{\alpha}(x) = \frac{1}{\sqrt{\Gamma(\alpha+1)}} x^{\alpha/2} e^{-x/2} /; , \psi_{-1}^{\alpha}(x) \equiv 0 .$$

These functions $\Psi_n^{\alpha}(x)$, called Gauss-Laguerre circular harmonic functions (CHFs), are referenced by integers n (referred by radial order) and α (referred by angular order). The real parts of $\Psi_n^{\alpha}(x)$ $(n = 0, 1, ..., 4; \alpha = 1, 2, ..., 5)$ are illustrated in Fig. 1.

The Gauss-Laguerre CHFs are self-steerable, i.e. they can be rotated by the angle θ using multiplication by the factor $e^{i\alpha\theta}$. They also keep their shape invariant under Fourier transformation. And they are suitable for multiscale and multicomponent image analysis [6], [9].

Let us consider an observed image $I(x, y)$ defined on the real plane R^2. Due to the orthogonality of Ψ_n^{α} family the image $I(x, y)$ can be expanded in the analysis point x_0, y_0 for fixed σ in Cartesian system as follows:

$$I(x_0, y_0) = \sum_{\alpha=-\infty}^{\infty} \sum_{n=0}^{\infty} g_{\alpha,n}(x_0, y_0; \sigma)\Psi_n^{\alpha}(\rho, \omega; \sigma) ,$$

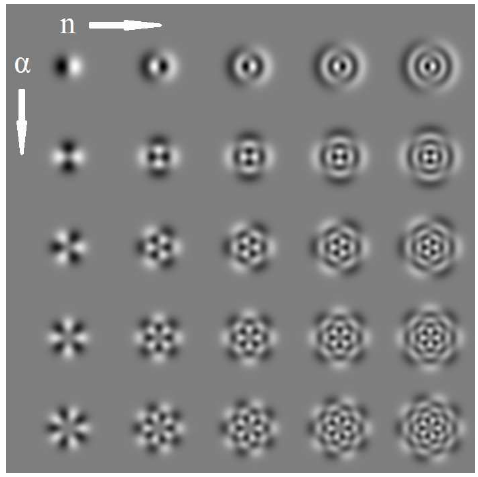

Fig. 1. The real part of Ψ_n^α $(n = 0, 1, ..., 4; \alpha = 1, 2, ..., 5)$.

where

$$g_{\alpha,n}(x_0, y_0; \sigma) = \int\limits_{-\infty}^{\infty}\int\limits_{-\infty}^{\infty} I(x_0, y_0)\overline{\Psi_n^\alpha(\rho, \omega; \sigma)}dxdy \ ,$$

and

$$\rho = \sqrt{(x - x_0)^2 + (y - y_0)^2}, \ \omega = \arctan(\frac{y - y_0}{x - x_0}) \ .$$

Let us consider the keypoints detection algorithm introduced in [6]. Let σ be the scale parameter and $\sigma \in [2^{-s_{\max}}, 2^{s_{\max}}]$ discretized in $(2s_{\max} + 1)$ octaves where each octave contains N_s uniformly sampled scales. So the set of scales is defined as $\{\sigma_j\}$, where $j = 0, 1, ..., 2N_s(2s_{\max} + 1) - 1$. Taking into account the Gauss-Laguerre CHFs property of being detectors for some image features (like edges, forks, crosses etc.), $n = 0, \alpha = 3, 4$ that corresponds to forks and crosses are considered. The set of $2N_s(2s_{\max} + 1)$ energy maps is defined as:

$$S(x, y; \sigma) = |g_{3,0}(x, y; \sigma)|^2 + |g_{4,0}(x, y; \sigma)|^2, \sigma \in \{\sigma_j\} \ ,$$

referred as image scalogram. The scalogram is inspected by 3D sliding window (5 x 5 x 3). The keypoints candidates $\overline{K} = (\overline{x}, \overline{y}; \overline{\sigma})$ are defined as the scalogram local maxima within the window. Here $(\overline{x}, \overline{y})$ is the keypoint coordinate and $\overline{\sigma}$ is the keypoint reference scale. So the image keypoints set is $\{\overline{K}\}$. This set is reduced by rejecting those keypoints $\overline{K}$ which have the same position $(\overline{x}, \overline{y})$ for more than two reference scales. And, finally, the keypoints $\overline{K}$ with energy value $S(\overline{x}, \overline{y}; \overline{\sigma})$ less than a selected threshold are omitted:

$$S(\overline{x}, \overline{y}; \overline{\sigma}) < T \cdot \max_{x,y}(S(x, y; \overline{\sigma})) \ . \qquad (1)$$

2.2 Gauss-Laguerre Keypoints Descriptors

The Gauss-Laguerre keypoints descriptors construction algorithm was first proposed in [6]. Each keypoint $\overline{K} = (\overline{x}, \overline{y}; \overline{\sigma})$ is associated to a local descriptor $\overline{\chi} = \{\overline{\chi}(n, \alpha, j)\}$. This is a complex-valued vector consisted of local image projections to a set of Gauss-Laguerre CHFs Ψ_n^α at $2j_{\max}$ scales neighbor to the keypoint $\overline{K}$ reference scale $\overline{\sigma}$. The $\overline{\chi}$ elements are defined as:

$$\overline{\chi}(n, \alpha, j) = \frac{g_{\alpha,n}(x, y; \sigma_j) \cdot e^{-i\alpha\theta_j}}{\|g_{\alpha,n}(x, y; \sigma_j) \cdot e^{-i\alpha\theta_j}\|} ,$$

$$n = 0, ..., n_{\max} , \alpha = 1, ..., \alpha_{\max} , j = -j_{\max}, ..., j_{\max} ,$$

where σ_j is the j-th scale following $\overline{\sigma}$ if $j > 0$, or preceding $\overline{\sigma}$ if $j < 0$ in the discretized scale space. The normalization makes descriptor invariant to the contrast changes. The phase shift $e^{-i\alpha\theta_j}$ is used to make the descriptors invariant to the keypoint pattern orientation, where

$$\theta_j = \arg(g_{1,0}(\overline{x}, \overline{y}; \sigma_j) .$$

The matching performance of this technique was demonstrated in [6] in comparison with SIFT algorithm. It was found in [6] that Gauss-Laguerre keypoints extraction method matching results overcome SIFT algorithm results in the case of rotation, scale and translation transformation of images. Nevertheless the computational cost of the algorithm is high.

2.3 Descriptors Computation using 2D Hermite Functions Expansion

The 2D Hermite functions $\Phi_{m,n}(x, y; \sigma)$ form the complete orthonormal system in L_2 space and can be defined as:

$$\Phi_{m,n}(x, y; \sigma) = \frac{1}{\sigma} \phi_m\left(\frac{x}{\sigma}\right) \phi_n\left(\frac{y}{\sigma}\right) , \phi_n(x) = \frac{(-1)^n e^{-\frac{x^2}{2}}}{\sqrt{2^n n! \sqrt{\pi}}} \cdot H_n(x) , \quad (2)$$

where $n = 0, 1, 2...$ and $H_n(x)$ are Hermite polynomials:

$$H_n(x) = (-1)^n e^{x^2} \frac{d}{dx^n} (e^{-x^2}) .$$

The Hermite functions $\phi_n(x)$ can be calculated using the following recurrence relations:

$$\phi_n(x) = x\sqrt{\frac{2}{n}} \phi_{n-1}(x) - \sqrt{\frac{n-1}{n}} \phi_{n-2}(x) , n = 2, 3, ...,$$

$$\phi_0(x) = \frac{1}{\sqrt[4]{\pi}} e^{-\frac{x^2}{2}} , \ \phi_1(x) = \frac{\sqrt{2}x}{\sqrt[4]{\pi}} e^{-\frac{x^2}{2}} .$$

The 2D Hermite image $I(x, y)$ expansion in the analysis point x_0, y_0 for fixed σ can be defined as:

$$I(x_0, y_0) = \sum_{m=0}^{\infty} \sum_{n=0}^{\infty} h_{m,n}(x_0, y_0; \sigma) \Phi_{m,n}(x, y; \sigma) \ ,$$

where

$$h_{m,n}(x_0, y_0; \sigma) = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} I(x_0 + x, y_0 + y) \Phi_{m,n}(x, y; \sigma) dx dy \ . \tag{3}$$

As one can see from (2), $\Phi_{m,n}(x, y; \sigma)$ functions are Cartesian separable, so the computation of (3) can be performed as follows:

$$\overline{h}_{m,n}(x_0, y_0; \sigma) = \int_{-\infty}^{\infty} I(x_0 + x, y) \phi_m(\frac{x}{\sigma}) dx \ , \tag{4}$$

for every fixed y and after that

$$h_{m,n}(x_0, y_0; \sigma) = \frac{1}{\sigma} \int_{-\infty}^{\infty} \overline{h}_{m,n}(x_0, y_0 + y; \sigma) \phi_n(\frac{y}{\sigma}) dy \ . \tag{5}$$

The coefficients of the 2D Hermite and Gauss Laguerre CHFs expansions are block-wise linearly related [10], [11] (the example of connection between Gauss-Laguerre circular harmonic functions and 2D Hermite functions is illustrated in Fig. 2). So the corresponding coefficients $g_{\alpha,n}$ and $h_{m,n}$ of image expansion to the sets of these functions are connected with the same relation.

Using the separability of $\Phi_{m,n}$ functions and interconnection between $\Phi_{m,n}$ and Ψ_n^α functions the number of operations for $g_{\alpha,n}$ computation can be reduced up to several times.

To suppress the descriptor changes due to the brightness changes we introduce the following step. Before expanding the image in keypoint neighborhood into the set of Gauss-Laguerre CHFs the average value of keypoints boundary pixels intensity is subtracted from keypoint neighborhood image intensity values.

Further acceleration can be achieved using fast Hermite projection method to compute coefficients $\overline{h}_{m,n}$ and $h_{m,n}$ in 1D expansions (4), (5).

3 Fast Hermite Projection Method

In common case 1D Hermite projection method is defined as:

$$f(x) = \sum_{m=0}^{\infty} c_m \phi_m(x)$$

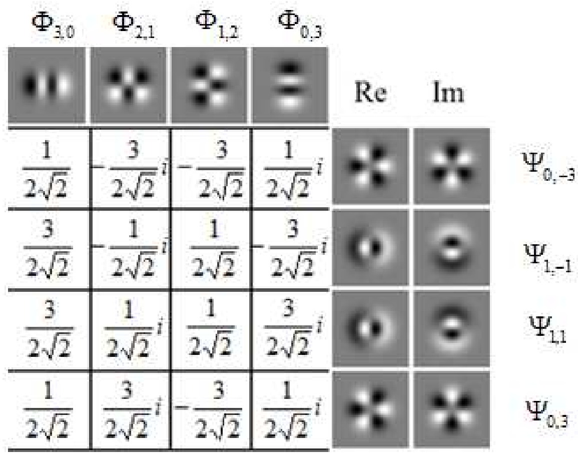

Fig. 2. An example of relation between Gauss-Laguerre circular harmonic functions and 2D Hermite functions

where $\phi_m(x)$ are 1D Hermite functions, c_m are Hermite coefficients:

$$c_m = \int_{-\infty}^{\infty} f(x)\phi_m(x)dx \; . \tag{6}$$

Each coefficient in (6) can be rewritten through Hermite polynomials as follows:

$$c_m = \frac{1}{\beta_m} \int_{-\infty}^{\infty} e^{-x^2} \left(f(x) e^{\frac{x^2}{2}} \right) H_m(x) dx \; ,$$

where $H_m(x)$ is Hermite polynomial, β_m is Hermite normalization constant:

$$\beta_m = \sqrt{2^m m! \sqrt{\pi}} \; .$$

This integral can be approximated by Gauss-Hermite quadrature [13]:

$$c_m = \frac{1}{\beta_m} \int_{-\infty}^{\infty} e^{-x^2} \left(f(x) e^{\frac{x^2}{2}} \right) H_m(x) dx \approx$$

$$\approx \frac{1}{\beta_m} \sum_{k=1}^{N} A_k \left(f(x_k) e^{\frac{x_k^2}{2}} \right) H_m(x_k) \; ,$$

where x_k – Hermite polynomials $H_N(x)$ zeros, A_k – associated weights:

$$A_k = \frac{2^{N-1} N! \sqrt{\pi}}{N^2 H_{N-1}^2(x_k)} \; .$$

Computation cost and precision loss of these associated weights increase with the increase of N [12]. This problem can be solved by replacement of Hermite polynomials by Hermite functions in (7) [12]. After simplification the following formula can be obtained:

$$c_m \approx \frac{1}{N} \sum_{k=1}^{N} \mu_{N-1}^{m}(x_k) f(x_k) \ ,$$

where $\mu_{N-1}^{m}(x_k)$ is an array of associated constants:

$$\mu_{N-1}^{m}(x_k) = \frac{\phi_m(x_k)}{\phi_{N-1}^{n}(x_k)} \ .$$

More details on fast Hermite projection method can be found in [12].

Keypoints descriptors elements computation can be even more accelerated using fast Hermite projection method to calculate $\overline{h}_{m,n}$ and $h_{m,n}$ in (4) and (5). However fast Hermite Projection method is lossy. So this acceleration brings in some error to the Gauss-Laguerre image expansion coefficients $g_{\alpha,n}$ and as a consequence keypoints descriptors elements.

4 Results

Proposed keypoints extraction algorithm has been tested on the images selected from the dataset freely available on the web, which provides the image and the relating homographies sequences (`http://www.robots.ox.ac.uk/~vgg/research/affine/`).

Typical values of achieved acceleration of initial descriptor construction algorithm are demonstrated in Table 1. The threshold T in keypoints detection (1) was set for each image independently to get about 1000 keypoints per image. The values of descriptors construction parameters were $n = 5$, $\alpha = 5$, $j_{\max} = 2$. Fast Hermite projection method was applied for keypoints with reference scale $\sigma > 5$. This value was chosen experimentally to get optimal balance between acceleration and approximation errors.

Table 1. Method acceleration results

Image name	2D Hermite separability acceleration	Fast Hermite projection method acceleration	Overall acceleration
boat1	3.77	1.42	5.36
boat2	3.80	1.45	5.50
boat3	3.82	1.39	5.31
graf1	1.44	3.22	4.67
graf2	1.49	3.25	4.85
graf3	1.49	3.37	5.02

The complete comparison of computational cost of proposed acceleration of Gauss-Laguerre descriptors construction algorithm and SIFT descriptors construction algorithm [5] is not given in this paper due to the fact that current implementation of Gauss-Laguerre keypoints descriptors construction algorithm is not optimized. Current implementation of the Gauss-Laguerre algorithm with the fast Hermite projection method acceleration is ~ 10.5 times slower than implementation of SIFT which is freely available on the web (`http://www.robots.ox.ac.uk/~vgg/research/affine/`).

The proposed method was compared in precision-recall [8] with SIFT keypoints descriptors construction algorithm. Descriptors were constructed for the same set of keypoints [5] selected with Gauss-Laguerre keypoints detection algorithm. Threshold T was identical for both pair images and its value was set to get at least 1000 keypoints in both images. The values of Gauss-Laguerre descriptors construction parameters were $n = 5$, $\alpha = 5$, $j_{\max} = 2$. Fast Hermite projection method was applied for keypoints with reference scale $\sigma > 5$. Different recall values were obtained changing the nearest neighbor distance ratio parameter in descriptors matching procedure proposed by Lowe [5].

Typical results are illustrated in Fig. 3, 4. The proposed method needs additional enhancement and optimization. Nevertheless even preliminary investigation showed promising results.

In Fig. 3 the results for graf1-graf2 image pair are given. This pair corresponds to points of view changing transformation. The obtained results show that Gauss-Laguerre descriptors and fast modification of Gauss-Laguerre descriptors perform better matching than SIFT descriptors for the same level of recall. However SIFT descriptors allow to reach the higher level of recall.

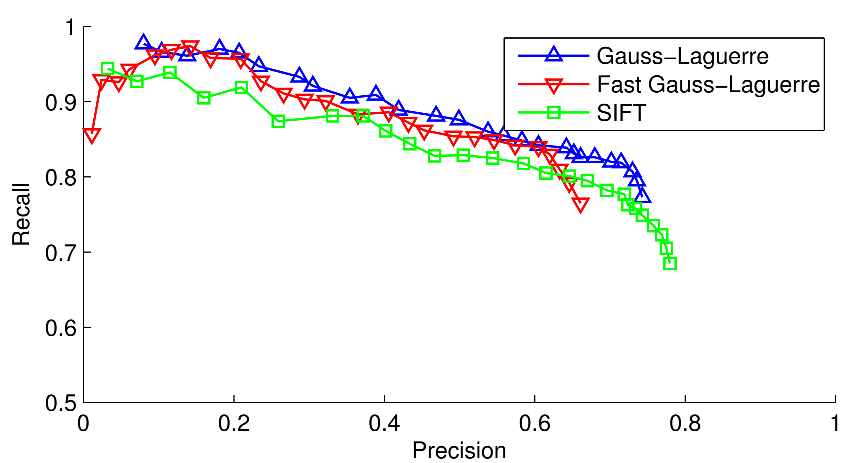

Fig. 3. Precision-Recall graph with different descriptors for graf1-graf2 image pair.

In Fig. 4 the results for boat1-boat2 image pair are given. This pair corresponds to rotation and zoom transformations. The obtained results show that Gauss-Laguerre descriptors performs better matching than SIFT descriptors for some levels of recall, but SIFT descriptors outperforms proposed descriptors in the area of high values of recall. Hermite projection based Gauss-Laguerre descriptors demonstrate less level of both recall and precision than SIFT and Gauss-Laguerre descriptors.

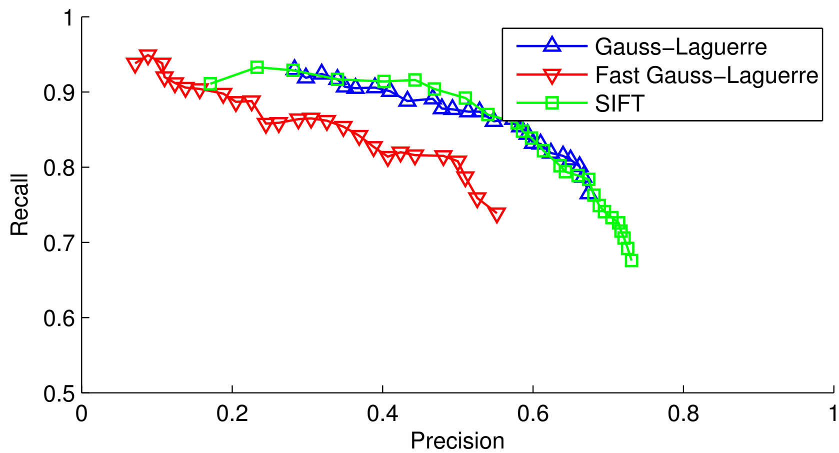

Fig. 4. Precision-Recall graph with different descriptors for boat1-boat2 image pair.

5 Conclusion

The efficient computation technique of Gauss-Laguerre keypoints descriptors using both the interconnection between Laguerre-Gauss circular harmonic functions and 2D Hermite functions and fast Hermite projection method have been proposed. The preliminary test results look promising. Nevertheless the tests showed that proposed descriptors are not fully invariant to brightness and contrast changes. Future work will include investigation in the field of brightness and contrast invariance of the descriptors and further improvement of Gauss-Laguerre keypoints detection algorithm.

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