Image inpainting
Outline

• Why inpaint?
• Image filtering (convolution), simplest inpainting
• Heat flow, diffusion process, diffusion equation
• Anisotropic diffusion
• Further development

Partly taken from “Restoring profanity” by Carola Schönlieb
Inpainting applications

• Image restoration
• Object removal
• Text removal
• Special effects

Image courtesy Andrea Baczyński
Simplest inpainting

- Image is 2D discrete function
- Form an iterative algorithm
- Each update gets you closer to the desired result

\[ I_{i,j}^{n+1} = I_{i,j}^n + \Delta t I_{t,i,j}^n \quad \forall (i, j) \in \Omega \]
Simplest inpainting

- Smooth image (using Gaussian filter)
- Replace missing pixels by smoothed version
- Repeat until the result stops changing

(MATLAB demo)
Convolution

Convolution of two functions \( f(x) \) and \( g(x) \)

\[
h(x) = f(x) \otimes g(x) = \int_{-\infty}^{+\infty} f(r) g(x-r) dr
\]

Discrete image processing 2D form

\[
H(x, y) = \sum_{j=1}^{\text{height}} \sum_{i=1}^{\text{width}} I(i, j) M(x-i, y-j)
\]

- **Output filtered image**
- **Image**
- **Convolution operator**
- **Filter (mask/kernel)**
- **Support region of filter where \( g(x-r) \) is nonzero**

Compute the convolution where there are valid indices in the kernel
Convolution example in 1D

Horizontal slice from Mandrill image

1D Gaussian filter

Filtered Signal

Image Processing using Matlab
Sumitha Balasuriya
Common convolution kernels

<table>
<thead>
<tr>
<th>Arithmetic mean filter (smoothing)</th>
<th>Laplacian (enhance edges)</th>
<th>Sharpening filter</th>
<th>Gaussian filter (smoothing)</th>
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</thead>
<tbody>
<tr>
<td>&gt;&gt;fspecial('average')</td>
<td>&gt;&gt;fspecial('laplacian')</td>
<td>&gt;&gt;fspecial('unsharp')</td>
<td>&gt;&gt;fspecial('gaussian')</td>
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Sobel operators (edge detection in x and y directions)
>>fspecial('sobel')
>>fspecial('sobel')'

The median filter is used for noise reduction. It works by replacing a pixel value with the median of its neighbourhood pixel values (vs the mean filter which uses the mean of the neighbourhood pixel values). Apply Matlab’s median filter function medfilt2 on the Mandrill and Lena images. Remember to use different filter sizes (3x3, 9x9, 16x16).
Useful functions for convolution

- Generate useful filters for convolution
  
  \[
  \text{fspecial('gaussian',[\text{kernel\_height} \ \text{kernel\_width}],\text{sigma})}
  \]

- 1D convolution
  
  \[
  \text{conv(signal,filter)}
  \]

- 2D convolution
  
  \[
  \text{conv2(double(I(:,:,2)),fspecial('gaussian',\text{[kernel\_height kernel\_width]},\text{sigma}),'valid')}\]

Perform the convolution of an image using Gaussian kernels with different sizes and standard deviations and display the output images.
Heat (diffusion) equation

- 1822 Joseph Fourier: heat propagation in rod

\[
\frac{\partial u(x, t)}{\partial t} = a^2 \frac{\partial^2 u(x, t)}{\partial x^2}
\]

- Same in image (2D)

\[
I_t(t, x, y) = a^2 (I_{xx} + I_{yy}) = a^2 \Delta I
\]

- Model intensity propagation as diffusion
Diffusion equation solution

• Left: initial distribution, right: t=10

• Damaged image:
Anisotropic diffusion

- Problem: edges are not preserved
- Do as professional restorator: continue along edges
- Solution: use anisotropic diffusion equation
Anisotropic diffusion (2)

- Fill the hole by isophote direction

\[ \vec{N} = \left[ -I_y, I_x \right] \]

- Anisotropic equation

\[ I_t = \nabla (\Delta I) \cdot \vec{N} \]
Since 1699, when French explorers landed at the great bend of the Mississippi River and celebrated the first Mardi Gras in North America, New Orleans has brewed a fascinating melange of cultures. It was French, then Spanish, then French again, then sold to the United States. Through all these years, and even into the 1900s, others arrived from everywhere: Acadians (Cajuns), Africans, indige-
Example: photo restoration
Further development

- Higher order equations for curvature minimization (smooth contour joining)
- Texture synthesis for large inpainting areas
- Combined approaches